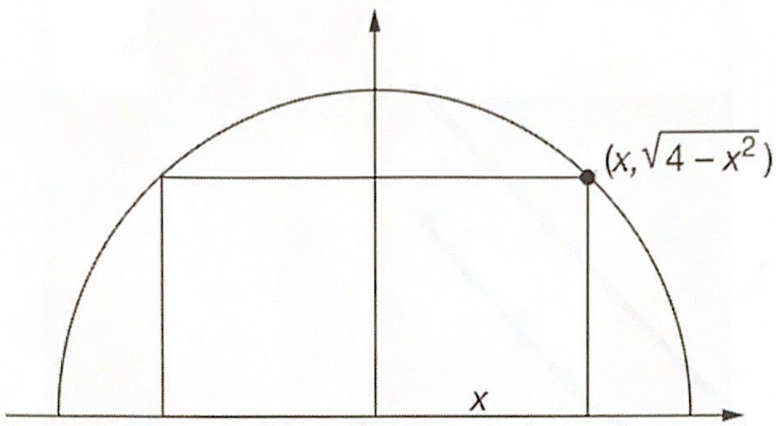
Practice 5.6 MC Optimization

Name

1.



The figure above shows a rectangle inscribed in a semicircle with a radius of 2. The area of such a rectangle is given by $A(x) = 2x\sqrt{4-x^2}$, where the width of the rectangle is $2x$. It can be shown that $A'(x) = \frac{-2x^2}{\sqrt{4-x^2}} + 2\sqrt{4-x^2}$ and A has critical values of -2, $-\sqrt{2}$, $\sqrt{2}$, and 2. It can also be shown that $A'(x)$ changes from positive to negative at $x = \sqrt{2}$. Which of the following statements is true?

(A) The inscribed rectangle with maximum area has dimensions $\sqrt{2}$ by $\sqrt{2}$.

(B) The inscribed rectangle with minimum area has dimensions $\sqrt{2}$ by $\sqrt{2}$.

(C) The inscribed rectangle with maximum area has dimensions $2\sqrt{2}$ by $\sqrt{2}$.

(D) The inscribed rectangle with minimum area has dimensions $2\sqrt{2}$ by $\sqrt{2}$.

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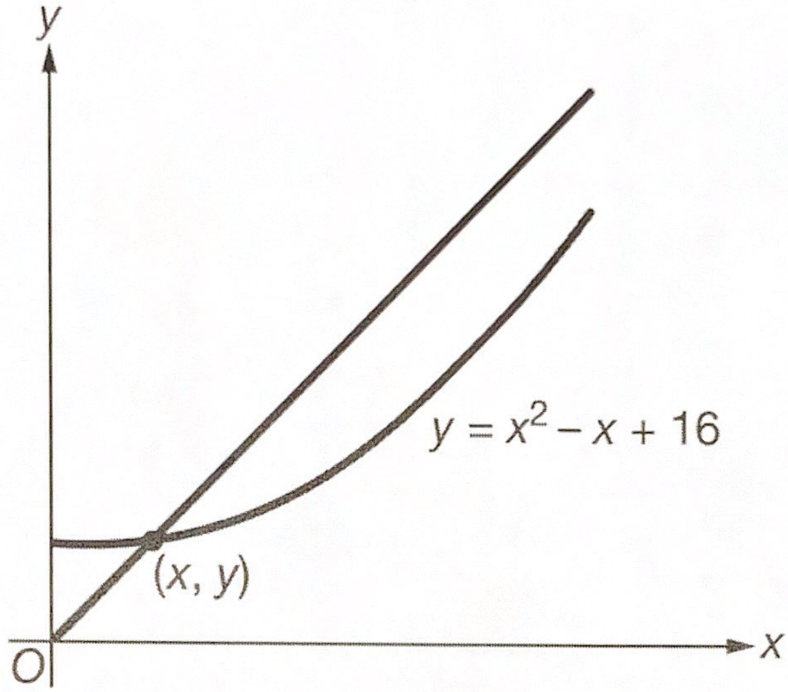
2.



Consider all lines in the xy-plane that pass through both the origin and a point (x, y) on the graph of $y = x^2 - x + 16$ for $1 \le x \le 8$. The figure above shows one such line and the graph of $y = x^2 - x + 16$. Which of the following statements is true?

(A) The line with minimum slope passes through the graph of $y = x^2 - x + 16$ at $x = 1$.

(B) The line with minimum slope passes through the graph of $y = x^2 - x + 16$ at $x = 4$.

(C) The line with minimum slope passes through the graph of $y = x^2 - x + 16$ at $x = 7$.

(D) The line with minimum slope passes through the graph of $y = x^2 - x + 16$ at $x = 8$.

~~$y = mx$~~ ~~$y = x^2 - x + 16$~~
~~$mx = x^2 - x + 16$~~
~~$-16 = x^2 - x - mx$~~
~~$0 = 2x - 1 - m$~~
~~$2x = 1 + m$~~
~~$x = \frac{1+m}{2}$~~

$y = mx$ $y = x^2 - x + 16$

$m = \frac{y}{x}$

$m = \frac{x^2 - x + 16}{x}$ $x = 0$

$m' = \frac{(2x-1)(x) - (x^2 - x + 16)}{x^2}$

$2x^2 - x - x^2 + x - 16$

$x^2 - 16 = 0$

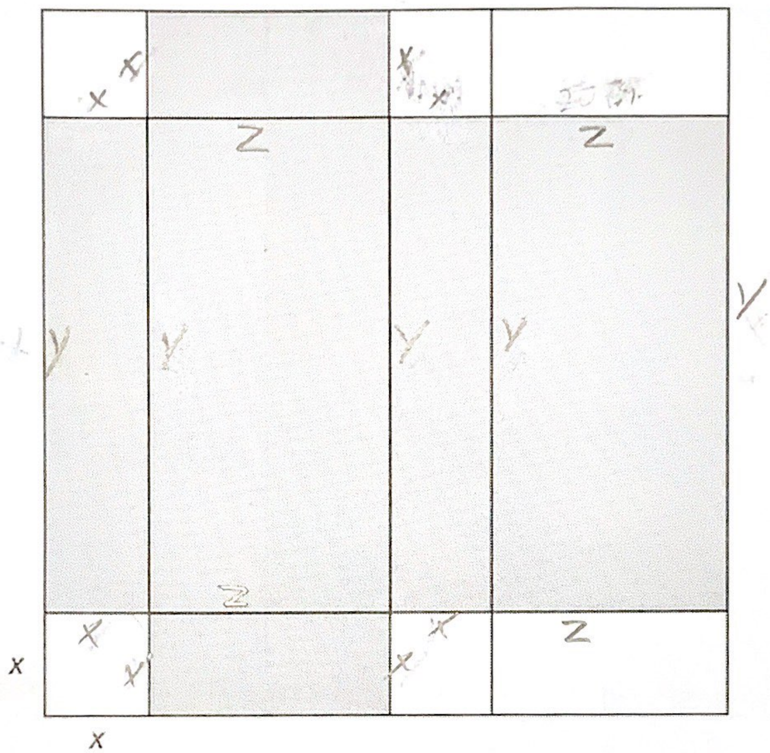
$x = 4$

+ − 4

NEG POS

Practice 5.6 MC Optimization

3.



The figure above represents a square sheet of cardboard with side length 20 inches. The sheet is cut and pieces are discarded. When the cardboard is folded, it becomes a rectangular box with a lid. The pattern for the rectangular box with a lid is shaded in the figure. Four squares with side length x and two rectangular regions are discarded from the cardboard. Which of the following statements is true? (The volume V of a rectangular box is given by $V = lwh$.)

(A) When $x = 10$ inches, the box has a minimum possible volume.

(B) When $x = 10$ inches, the box has a maximum possible volume.

(C) When $x = \frac{10}{3}$ inches, the box has a minimum possible volume.

(D) When $x = \frac{10}{3}$ inches, the box has a maximum possible volume.

$V = xy\cancel{z}$

$\cancel{SA = 2yz + 2yx + 2xz}$

$y = 20 - 2x$ $\quad 20 = 2z + 2x$

$2z = 20 - 2x$

$V = x(20-2x)(10-x)$ $\quad z = 10 - x$

$V = x(200 - 20x - 20x + 2x^2)$

$V = x(200 - 40x + 2x^2)$

$V = 200x - 40x^2 + 2x^3$

$V' = 200 - 80x + 6x^2$

$0 = 200 - 80x + 6x^2$

$0 = 2(100 - 40x + 3x^2)$

$(3x - 10)(x - 10)$

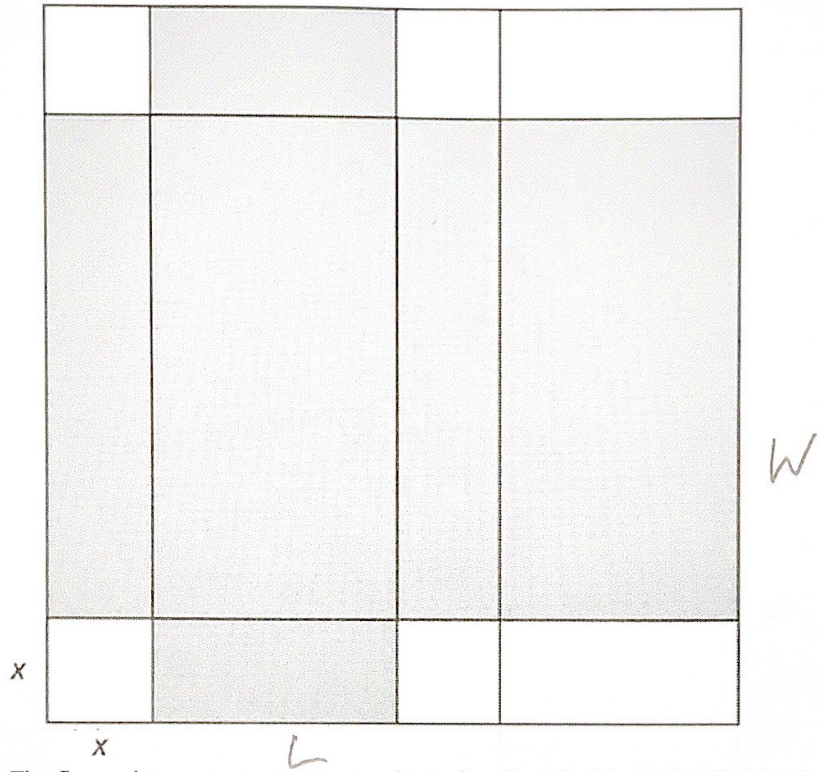
$x = \frac{10}{3}$ $\quad x = 10$

$- -$ $\frac{10}{3}$ $+ +$ 10 $+ +$

POS NEG POS

Practice 5.6 MC Optimization

4.



The figure above represents a square sheet of cardboard with side length 40 inches. The sheet is cut and pieces are discarded. When the cardboard is folded, it becomes a rectangular box with a lid. The pattern for the rectangular box with a lid is shaded in the figure. Four squares with side length x and two rectangular regions are discarded from the cardboard. Which of the following statements is true? (The volume V of a rectangular box is given by $V = lwh$.)

(A) When $x = 20$ inches, the box has a minimum possible volume.

(B) When $x = 20$ inches, the box has a maximum possible volume.

(C) When $x = \frac{20}{3}$ inches, the box has a minimum possible volume.

(D) When $x = \frac{20}{3}$ inches, the box has a maximum possible volume.

$V = lwh$

$40 = 2L + 2x$

$2L = 40 - 2x$

$\boxed{L = 20 - x}$

$40 = W + 2x$

$\boxed{W = 40 - 2x}$

$V = (x)(20-x)(40-2x)$

$= x(800 - 40x - 40x + 2x^2)$

$= x(800 - 80x + 2x^2)$ $\quad V' = 800 - 160x + 6x^2$

$= 800x - 80x^2 + 2x^3$

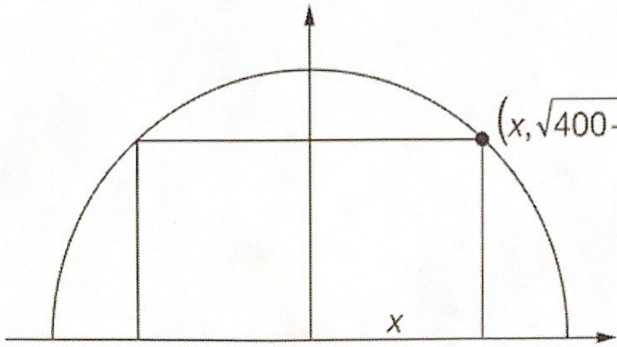
$V' = 2(400 - 80x + 3x^2)$

$(3x - 20)(x - 20)$

POS $\frac{20}{3}$ NEG 20

Practice 5.6 MC Optimization

5.



The figure above shows a rectangle inscribed in a semicircle with a radius of 20. The area of such a rectangle is given by $A(x) = 2x\sqrt{400-x^2}$, where the width of the rectangle is $2x$. It can be shown that $A'(x) = \frac{-2x^2}{\sqrt{400-x^2}} + 2\sqrt{400-x^2}$ and A has critical values of -20, $-10\sqrt{2}$, $10\sqrt{2}$, and 20. It can also be shown that $A'(x)$ changes from positive to negative at $x = 10\sqrt{2}$. Which of the following statements is true?

(A) The inscribed rectangle with maximum area has dimensions $10\sqrt{2}$ by $10\sqrt{2}$.

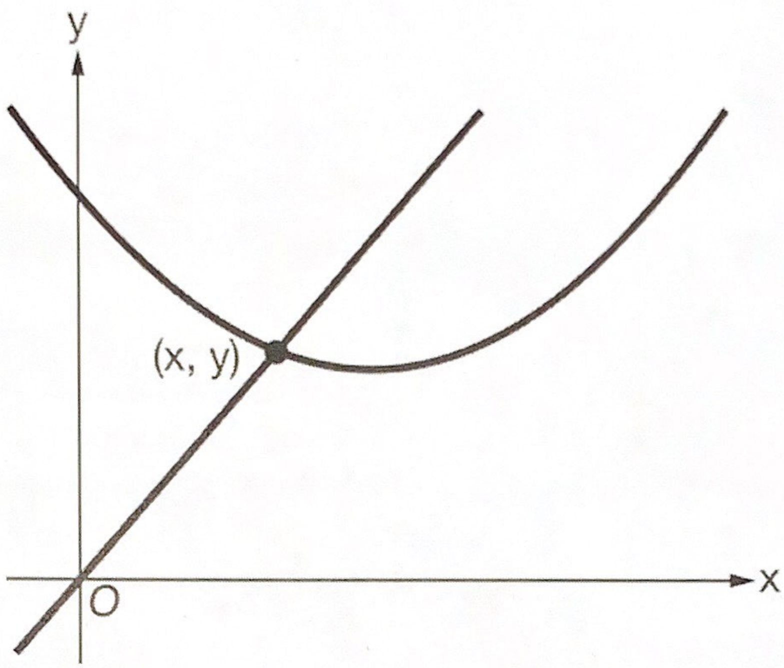
(B) The inscribed rectangle with minimum area has dimensions $10\sqrt{2}$ by $10\sqrt{2}$.

(C) The inscribed rectangle with maximum area has dimensions $20\sqrt{2}$ by $10\sqrt{2}$.

(D) The inscribed rectangle with minimum area has dimensions $20\sqrt{2}$ by $10\sqrt{2}$.

Practice 5.6 MC Optimization

6.



Consider all lines in the xy-plane that pass through both the origin and a point (x, y) on the graph of $y = x^2 - 4x + 9$ for $1 \le x \le 4$. The figure above shows one such line and the graph of $y = x^2 - 4x + 9$. Which of the following statements is true?

(A) The line with minimum slope passes through the graph of $y = x^2 - 4x + 9$ at $x = 1$.

(B) The line with minimum slope passes through the graph of $y = x^2 - 4x + 9$ at $x = 2$.

(C) The line with minimum slope passes through the graph of $y = x^2 - 4x + 9$ at $x = 3$.

(D) The line with minimum slope passes through the graph of $y = x^2 - 4x + 9$ at $x = 4$.

$y = mx$ $\quad$ $y = x^2 - 4x + 9$

$m = \frac{y}{x}$

$m = \frac{x^2 - 4x + 9}{x}$ $\quad$ $m' = 1 - \frac{9}{x^2}$

$m = (x^2 - 4x + 9)x^{-1}$ $\quad$ $m' = \frac{x^2 - 9}{x^2}$ $\quad$ $x = \pm 3$

$m = (x - 4 + 9x^{-1})$

Practice FRQ 5.6

1.) a.) $G'(3) \approx \frac{19.5-18.5}{5-1} \approx \frac{1}{4} \approx 250$ gallons/hour.

b.) $ARC\ [0,10] = \frac{24-22}{10-0} = \frac{2}{10} = 0.2$ thousand gallons/hour.

- $G(t)$ is continuous on $[0,10]$
- $G(t)$ is differentiable on $(0,10)$

$\therefore$ The Mean Value Theorem Guarantees a value of t such that $G'(t)=0.2$ on the interval $(0,10)$.

c.) $y-19.5 = 0.5(x-5)$

$y-19.5 = 0.5(7-5)$

$y-19.5 = 0.5(2)$

$y-19.5 = 1$

$y = 20.5$

$\therefore G(7) = 20.5$

- $G(t)$ is concave up at $t=7$ because $G''(t)>0$ on $5 \le t \le 10$

$\therefore$ the tangent line at $G(5)$ is under $G(t)$

$\therefore$ this is an under approximation.

d.) $R(t) = \frac{100t}{t^2+4}$

$R'(t) = \frac{100(t^2+4)-(100t(2t))}{(t^2+4)^2}$

$R'(t) = \frac{100((t^2+4)-2t^2)}{(t^2+4)^2}$

$R'(t) = \frac{100(-t^2+4)}{(t^2+4)^2}$

$0 = -t^2+4$

$t^2 = 4$

$t = \pm 2$

$t = 2$

$(t^2+4)^2 = 0$

$t^2+4 = 0$

$\cancel{t^2 = -4}$

$R(0) = 0$

$R(10) = \frac{100(10)}{(10)^2+4} = \frac{1000}{104} = \frac{500}{57}$

$R(2) = \frac{100(2)}{(2)^2+4} = \frac{200}{8} = 25$

$\boxed{\therefore \text{ the rate that gasoline flows is at an absolute maximum at 2 hours.}}$

② a) $A'(10) = \frac{25-18}{15-5} = \frac{7}{10}$ gallons/hour

b) ARC on $[0,30] = \frac{16-10}{30-0} = \frac{6}{30} = \frac{1}{5}$

- $A(t)$ is continuous on $[0,30]$
- $A(t)$ is differentiable on $(0,30)$

∴ The MVT Guarantees a value of t such that $A'(t) = \frac{1}{5}$ on $[0,30]$

c) $G(t) = 5t - \frac{2}{3}(t+9)^{\frac{3}{2}} + 28$

$G'(t) = 5 - (t+9)^{\frac{1}{2}}$

$0 = 5 - (t+9)^{\frac{1}{2}}$

$(t+9)^{\frac{1}{2}} = 5$

$t+9 = 25$

$\boxed{t = 16}$

MAX

0 POS 16 NEG 35

∴ the number of gallons of olive oil in the tank is at a maximum at $t = 16$ hours.

d) $G'(7) = 5 - (7+9)^{\frac{1}{2}}$
$= 5 - (16)^{\frac{1}{2}}$
$= 5 - 4$
$= 1$

$G(7) = 5(7) - \frac{2}{3}(7+9)^{\frac{3}{2}} + 28$
$= 35 - \frac{2}{3}(16)^{\frac{3}{2}} + 28$
$= 35 - \frac{2}{3}(64) + 28$
$= 35 - \frac{128}{3} + 28$
$= \frac{105}{3} - \frac{128}{3} + \frac{84}{3}$
$= -\frac{23}{3} + \frac{84}{3}$
$\boxed{= \frac{61}{3}}$

$35 \times 3 = 105$

$28 \times 3 = 84$

$y - \frac{61}{3} = x - 7$

$y - \frac{61}{3} = 8 - 7$

$y - \frac{61}{3} = \frac{3}{3}$

$\boxed{y = \frac{64}{3}}$

$g'(t) = 5 - (t+9)^{\frac{1}{2}}$

$g''(t) = -\frac{1}{2}(t+9)^{-\frac{1}{2}}$

$g''(7) = -\frac{1}{2}(7+9)^{-\frac{1}{2}}$
$= -\frac{1}{2}(16)^{-\frac{1}{2}}$
$= -\frac{1}{2(4)}$
$= -\frac{1}{8}$

∴ $g''(7)$ is negative
∴ $g(t)$ is concave down at $t = 7$
∴ this is an over approximation.