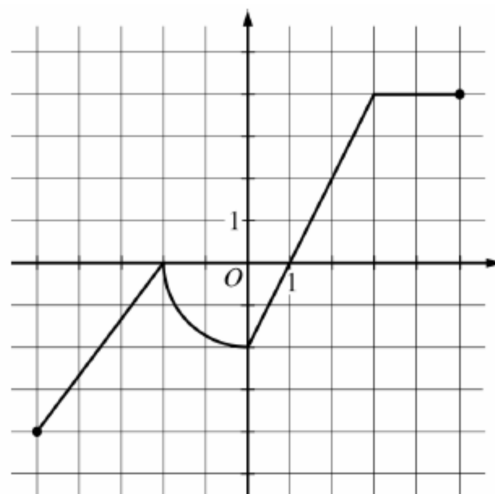


NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

Graph of  $f$ 

3. The graph of the function  $f$ , consists of three line segments and a quarter of a circle, is shown above. Let  $g$  be the function defined by  $g(x) = \int_1^x f(t) dt$ .
- (a) Find the average rate of change of  $g$  from  $x = -5$  to  $x = 5$ .

- (b) Find the instantaneous rate of change of  $g$  with respect to  $x$  at  $x = 3$ , or state that it does not exist.

(c) On what open intervals, if any, is the graph of  $g$  concave up? Justify your answer.

(d) Find all  $x$ -values in the interval  $-5 < x < 5$  at which  $g$  has a critical point as the location of a local minimum, or local maximum, or neither. Justify your answers.

NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

4. Consider the curve given by the equation  $2(x - y) = 3 + \cos y$ . For all points on the curve,  $\frac{2}{3} \leq \frac{dy}{dx} \leq 2$ .

(a) Show that  $\frac{dy}{dx} = \frac{2}{2 - \sin y}$ .

- (b) For  $-\frac{\pi}{2} < y < \frac{\pi}{2}$ , there is a point  $P$  on the curve through which the line tangent to the curve has slope 1. Find the coordinates of the point  $P$ .

(c) Determine the concavity of the curve at points which  $-\frac{\pi}{2} < y < \frac{\pi}{2}$ . Give a reason for your answer.

(d) Let  $y = f(x)$  be a function, defined implicitly by  $2(x - y) = 3 + \cos y$ , that is continuous on the closed interval  $[2, 2.1]$  and differentiable on the open interval  $(2, 2.1)$ . Use the Mean Value Theorem on the interval  $[2, 2.1]$  to show that  $\frac{1}{15} \leq f(2.1) - f(2) \leq \frac{1}{5}$ .