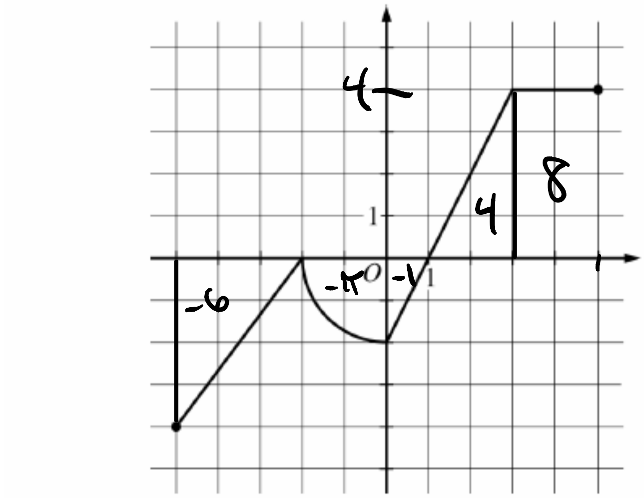
NO CALCULATOR IS ALLOWED FOR THIS QUESTION.



Graph of f

3. The graph of the function f, consists of three line segments and a quarter of a circle, is shown above. Let g be the function defined by $g(x) = \int_1^x f(t)dt$.

(a) Find the average rate of change of g from $x = -5$ to $x = 5$.

$$ARC = \frac{g(-5) - g(5)}{-5 - 5} \quad +1$$

$$= \frac{(\pi+7)-12}{-10}$$

(Answer does ~~NOT~~ need simplified)

$$ARC = \frac{\pi - 5}{-10} \quad +2$$

SIDE WORK

$$g(-5) = \int_1^{-5} f(t)dt = -(-7-\pi) = \pi + 7$$

$$g(5) = \int_1^{5} f(t)dt = 12$$

(b) Find the instantaneous rate of change of g with respect to x at $x = 3$, or state that it does not exits.

$$g'(x) = f(x) \cdot x'$$

$$g'(3) = f(3) = 4$$

The instantaneous rate of change of g at $x=3$ is 4 +1

(c) On what open intervals, if any, is the graph of g concave up? Justify your answer.

$g'(x) = f(x)$ is increasing on $-5 < x < -2$ and $0 < x < 3$ +1

$\therefore$ The graph of g is concave up on $-5 < x < -2$ and $0 < x < 3$ +1

(d) Find all $x-$values in the interval $-5 < x < 5$ at which g has a critical point as the location of a local minimum, or local maximum, or neither. Justify your answers.

+1 • $g'(x) = f(x) = 0$ at $x = -2$ and $x = 1$

+1 • $g'(x)$ does not change sign at $x = -2$

$\therefore$ $g(x)$ has neither a local maximum or local minimum at $x = -2$

+1 • $g'(x)$ changes from negative to positive at $x = 1$

$\therefore$ $g(x)$ has local minimum at $x = 1$

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4. Consider the curve given by the equation $2(x-y) = 3 + \cos y$. For all points on the curve, $\frac{2}{3} \le \frac{dy}{dx} \le 2$.

(a) Show that $\frac{dy}{dx} = \frac{2}{2-\sin y}$.

$$2(x-y) = 3 + \cos y$$

$$2x - 2y = 3 + \cos y$$

$$\frac{d}{dx}(2x) - \frac{d}{dx}(2y) = \frac{d}{dx}(3) + \frac{d}{dx}(\cos y)$$

$$2 - 2\cdot\frac{dy}{dx} = 0 - \sin y \cdot \frac{dy}{dx} \quad +1$$

$$2 = 2\frac{dy}{dx} - \sin y \frac{dy}{dx}$$

$$2 = \frac{dy}{dx}(2 - \sin y)$$

$$\frac{2}{2-\sin y} = \frac{dy}{dx} \quad +1$$

(b) For $-\frac{\pi}{2} < y < \frac{\pi}{2}$, there is a point P on the curve through which the line tangent to the curve has slope 1. Find the $\boxed{\text{coordinates of the point } P}$

$$\frac{dy}{dx} = 1 \quad +1$$

$$\frac{2}{2-\sin y} = 1$$

$$2 = 2 - \sin y$$

$$0 = -\sin y$$

$$0 = \sin y$$

$$0 = y$$

$\boxed{y=0}$

$$2(x-y) = 3 + \cos y$$

$$2(x-0) = 3 + \cos 0$$

$$2x = 3 + 1$$

$$2x = 4$$

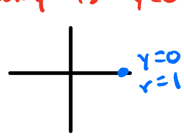
$$x = 2$$

P has coordinates $(2,0)$ +1

NOT NEEDED

why is $\underbrace{y=0}_{\text{angle}}$ if $\sin y = 0$

angle



$\sin(\text{angle}) = \frac{y}{r} = \frac{0}{1}$

(c) Determine the concavity of the curve at points which $-\frac{\pi}{2} < y < \frac{\pi}{2}$. Give a reason for your answer.

$\frac{dy}{dx} = \frac{2}{2-\sin y}$

$\frac{d^2y}{dx^2} = \frac{(0)\cdot(2-\sin y) - 2(0-\cos y\frac{dy}{dx})}{(2-\sin y)^2}$

$\frac{d^2y}{dx^2} = \frac{2\cos y\frac{dy}{dx}}{(2-\sin y)^2}$ +2

$\frac{d^2y}{dx^2} = \frac{2\cdot(+)(+)}{(+)} = \text{positive}$

- $\frac{d^2y}{dx^2} > 0$ for all y on $-\frac{\pi}{2} < y < \frac{\pi}{2}$

$\therefore$ the curve is concave up on $-\frac{\pi}{2} < y < \frac{\pi}{2}$ +1

(d) Let $y = f(x)$ be a function, defined implicitly by $2(x-y) = 3 + \cos y$, that is continuous on the closed interval $[2, 2.1]$ and differentiable on the open interval $(2, 2.1)$. Use the Mean Value Theorem on the interval $[2, 2.1]$ to show that $\frac{1}{15} \le f(2.1) - f(2) \le \frac{1}{5}$.

+1 Mean value Theorem ...

A value of c on $(2, 2.1)$ must exist so that $f'(c) = \frac{f(2.1)-f(2)}{2.1-2}$

$\frac{2}{3} \le f'(x) \le 2$ [from stem of problem]

$\frac{2}{3} \le \frac{f(2.1)-f(2)}{0.1} \le 2$

+1

$\frac{2}{3} \le [f(2.1)-f(2)]\cdot 10 \le 2$

$\frac{2}{3}\cdot\frac{1}{10} \le f(2.1)-f(2) \le 2\cdot\frac{1}{10}$

$\frac{1}{15} \le f(2.1)-f(2) \le \frac{1}{5}$