CALC

2001 AP® CALCULUS AB Problem #2

t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t. The table above shows the water temperature as recorded every 3 days over a 15-day period.

(a) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.

$$W'(12) = \frac{W(9) - W(15)}{9 - 15} = \frac{24 - 21}{-6} = \frac{3}{-6} = -\frac{1}{2}\ ^\circ C/day$$

+1 +1

(b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \le t \le 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

+1

$$\int_0^{15} W(t)\,dt = \frac{1}{2}(3)\left[20 + 2(31) + 2(28) + 2(24) + 2(22) + 21\right]$$

$$= 376.5\ ^\circ C$$

$$\text{Average Temp} = \frac{1}{15-0}\int_0^{15} W(t)\,dt$$

$$= \frac{1}{15}(376.5)$$

$$= 25.1\ ^\circ C/day$$ +1

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(c) A student proposes the function P, given by $P(t) = 20 + 10te^{(-t/3)}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.

$P'(t) = (10)\cdot e^{-t/3} + 10t\cdot e^{-t/3}\cdot\left(-\frac{1}{3}\right)$

$P'(t) = 10e^{-t/3} - \frac{10}{3}te^{-t/3}$

$P'(12) = 10e^{-12/3} - \frac{10}{3}(12)e^{-12/3} = 10e^{-4} - 40e^{-4} = -30e^{-4}$

$P'(12) \approx -0.549\,^\circ C/\text{day}$ +1

At $t = 12$ days, the temperature of the water is decreasing by $0.549^\circ C$ per day. +1

(d) Use the function P defined in part (c) to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \le t \le 15$ days.

+1 $\frac{1}{15-0}\int_0^{15} P(t)\,dt = 25.757^\circ C$ +1 +1