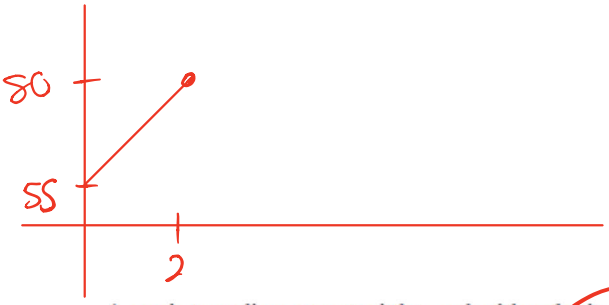
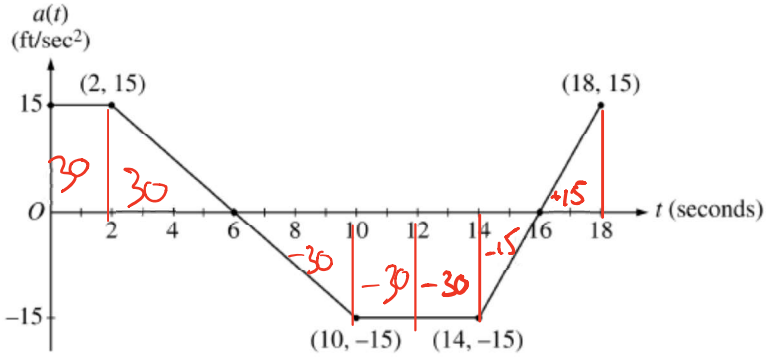
2001 AP® CALCULUS AB

Problem #3





$v(0) = 55$ ft/sec

A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \le t \le 18$ seconds, the car's acceleration $a(t)$, in ft/sec^2, is the piecewise linear function defined by the graph above.

(a) Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?

$v'(2) = a(2) = 15$. Since $a(2) = 15$, the acceleration is positive which implies the velocity is increasing at $t = 2$.

(b) At what time in the interval $0 \le t \le 18$, other than $t = 0$, is the velocity of the car 55 ft/sec? Why?

FTC

$$\int_0^b a(t)\,dt = v(b) - v(0)$$

$$= 55 - 55$$

$$\int_0^b a(t)\,dt = 0$$

(when does Area under curve sum to 0?)

$\therefore v(12) = 55$ ft/sec

(c) On the time interval $0 \le t \le 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.

$$\int_0^t a(t)\,dt = v(t) - v(0)$$

$$\int_0^t a(t)\,dt = v(t) - 55$$

$$v(t) = 55 + \int_0^t a(t)\,dt$$

CANDIDATES TEST

t	v(t)
0	55
6	55+60
16	55+(-45)
18	55+(-30)

∴ At 6 seconds, the car reaches a maximum velocity of 115 ft/s.

(d) At what times in the interval $0 \le t \le 18$, if any, is the car's velocity equal to zero? Justify your answer.

$$\int_0^b a(t)\,dt = v(b) - v(0)$$

$$= 0 - 55$$

$$\int_0^b a(t)\,dt = -55$$

The sum of the areas under the curve never sum to -55 ft/sec

OR

The car's velocity never reaches zero. The absolute minimum velocity on [0,18] is 10 ft/s when t = 16 seconds as shown in part (c).