Name____Date____________________Class___________

Homework 7.3

AP® CALCULUS AB **2001 Question 6**

The function f is differentiable for all real numbers. The point $\left(3, \frac{1}{4}\right)$ is on the graph of $y = f(x)$, and the slope at each point (x, y) on the graph is given by $\frac{dy}{dx} = y^2(6-2x)$.

(a) Find $\frac{d^2y}{dx^2}$ and evaluate it at the point $\left(3, \frac{1}{4}\right)$.

+2

$$\frac{dy}{dx} = y^2(6-2x)$$

$$\frac{d^2y}{dx^2} = 2y \cdot \frac{dy}{dx}(6-2x) + y^2(-2)$$

$$\frac{d^2y}{dx^2} = 2y \cdot \left[y^2(6-2x)\right](6-2x) - 2y^2$$

$$\frac{d^2y}{dx^2} = 2y^3(6-2x)^2 - 2y^2$$

$$\left.\frac{d^2y}{dx^2}\right|_{(3,1/4)} = 2\left(\tfrac{1}{4}\right)^3\left(6-2(3)\right)^2 - 2\left(\tfrac{1}{4}\right)^2$$

$$= 2\left(\tfrac{1}{64}\right)(6-6)^2 - 2\left(\tfrac{1}{16}\right)$$

$$= 0 - \tfrac{1}{8}$$

$$\left.\frac{d^2y}{dx^2}\right|_{(3,1/4)} = -1/8$$ +1

(b) Find $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = y^2(6-2x)$ with the initial condition $f(3) = \frac{1}{4}$.

$$\int y^{-2}\,dy = \int (6-2x)\,dx$$ +1

+1 $$-y^{-1} = 6x - x^2 + C$$ +1

$$-\frac{1}{y} = -x^2 + 6x + C$$

+1

at $(3, \frac{1}{4})$

$$\frac{-1}{\frac{1}{4}} = -(3)^2 + 6(3) + C$$

$$-4 = -9 + 18 + C$$

$$-4 = 9 + C$$

$$-13 = C$$ +1

$$\frac{-1}{y} = -x^2 + 6x - 13$$

$$-1 = (-x^2 + 6x - 13)y$$

$$\frac{-1}{-x^2 + 6x - 13} = y$$

$$f(x) = \frac{1}{x^2 - 6x + 13}$$ +1