Consider the differential equation $\frac{dy}{dx} = \frac{3-x}{y}$.

(a) Let $y = f(x)$ be the particular solution to the given differential equation for $1 < x < 5$ such that the line $y = -2$ is tangent to the graph of f. Find the x-coordinate of the point of tangency, and determine whether f has a local maximum, local minimum, or neither at this point. Justify your answer.

If $y=-2$ is tangent, then $\frac{dy}{dx}=0$ (horizontal tangent line)

$\frac{dy}{dx} = \frac{3-x}{y}$

$0 = \frac{3-x}{-2}$

$0 = 3-x$

$x = 3$ +1

POT $(3,-2)$

2nd Derivative Test

$\frac{d^2y}{dx^2} = \frac{(-1)(y)-(3-x)(1)\frac{dy}{dx}}{y^2}$

$\frac{d^2y}{dx^2} = \frac{-y-(3-x)\left(\frac{3-x}{y}\right)}{y^2}$

$\left.\frac{d^2y}{dx^2}\right|_{(3,-2)} = \frac{-(-2)-(3-3)\left(\frac{3-3}{-2}\right)}{(-2)^2}$

$= \frac{2-0}{4}$

$\left.\frac{d^2y}{dx^2}\right|_{(3,-2)} = \frac{1}{2}$ +1

Since $\frac{d^2y}{dx^2} > 0$ at $(3,-2)$ $f(x)$ is concave up which means $(3,-2)$ is a relative minimum of $f(x)$. +1

(b) Let $y = g(x)$ be the particular solution to the given differential equation for $-2 < x < 8$, with the initial condition $g(6) = -4$. Find $y = g(x)$.

$\frac{dy}{dx} = \frac{3-x}{y}$

$\int y\,dy = \int (3-x)\,dx$ +1

+1 $\frac{1}{2}y^2 = 3x - \frac{1}{2}x^2 + C$ +1

+1 at $(6,-4)$

$\frac{1}{2}(-4)^2 = 3(6) - \frac{1}{2}(6)^2 + C$

$\frac{1}{2}(16) = 18 - \frac{1}{2}(36) + C$

$8 = 18 - 18 + C$

$8 = C$ +1

$\frac{1}{2}y^2 = 3x - \frac{1}{2}x^2 + 8$

$y^2 = 6x - x^2 + 16$

$y = \pm\sqrt{6x - x^2 + 16}$

which one contains $(6,-4)$?

$y = -\sqrt{-x^2 + 6x + 16}$ +1