Calc

2004 AP® CALCULUS AB Problem #1

Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by

$$F(t) = 82 + 4\sin\left(\frac{t}{2}\right) \text{ for } 0 \le t \le 30,$$

where $F(t)$ is measured in cars per minute and t is measured in minutes.

(a) To the nearest whole number, how many cars pass through the intersection over the 30-minute period?

$\int_0^{30} \left[82 + 4\sin\left(\frac{t}{2}\right)\right] dt = 2474$ cars

+1 +1 +1

(b) Is the traffic flow increasing or decreasing at $t = 7$? Give a reason for your answer.

$F(t) = 82 + 4\sin\left(\frac{t}{2}\right)$ for $0 \le t \le 30$,

$F'(7) = -1.873$

$F'(x) < 0$ at $x = 7$

$\therefore$ traffic flow is decreasing at $x = 7$

+1

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(c) What is the average value of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

$$\frac{1}{15-10}\int_{10}^{15} F(t)\,dt = 81.899 \text{ cars/minute}$$

+1 +1 +1

(d) What is the average rate of change of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

$$ARC = \frac{F(10)-F(15)}{10-15} = 1.518 \text{ car/min}^2$$

+1

+1 Proper units in (c) and (d)