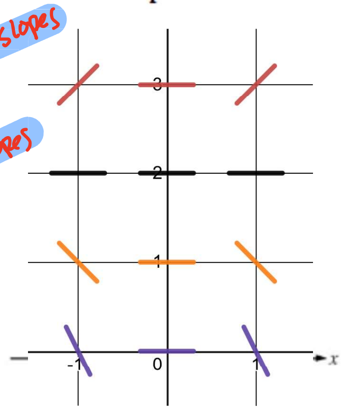
2004 AP® CALCULUS AB
Question 5 (Form B)

Consider the differential equation $\frac{dy}{dx} = x^4(y-2)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the test booklet.)

+1 for all other slopes

+1 for zero slopes



(x,y)	$x^4(y-2)$
$(0,y)$	0
$(x,2)$	0
$(\pm1,0)$	$(1)(-2) = -2$
$(\pm1,1)$	$(1)(-1) = -1$
$(\pm1,3)$	$(1)(1) = 1$

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are negative.

The slope will be negative when $x \neq 0$ and $y < 2$ for all (x,y) +1

$x^4(y-2) < 0$

$x^4 < 0$ and $y-2 < 0$

$x \neq 0$ and $y < 2$

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 0$.

$u = y-2$
$du = dy$

$\int \frac{1}{y-2}\,dy = \int x^4\,dx$ +1

+1 $\int \frac{1}{u}\,du = \frac{1}{5}x^5 + C$ +1

$\ln|u| = \frac{1}{5}x^5 + C$

$\ln|y-2| = \frac{1}{5}x^5 + C$

+1 $(0,0)$

$\ln|0-2| = \frac{1}{5}(0)^5 + C$

$\ln|-2| = C$

$\ln 2 = C$ +1

$\ln|y-2| = \frac{1}{5}x^5 + \ln 2$

$|y-2| = e^{\frac{1}{5}x^5 + \ln 2}$

$|y-2| = e^{\frac{1}{5}x^5}e^{\ln 2}$

$|y-2| = 2e^{\frac{1}{5}x^5}$

$y-2 = -2e^{\frac{1}{5}x^5}$ or $y-2 = 2e^{\frac{1}{5}x^5}$

$y = 2-2e^{\frac{1}{5}x^5}$ or $y = 2+2e^{\frac{1}{5}x^5}$

which one contains $(0,0)$?

$f(x) = 2-2e^{\frac{1}{5}x^5}$ +1