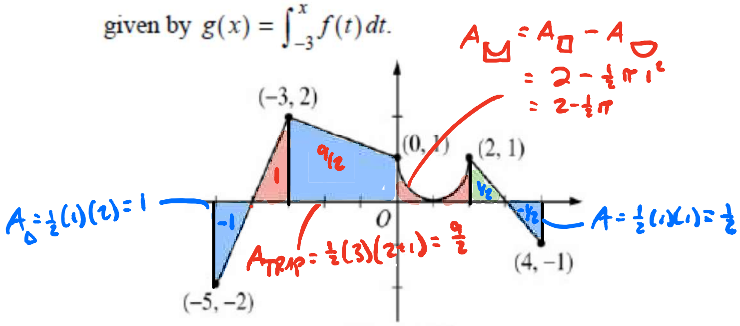
2004 AP® CALCULUS AB
Question 5

The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^{x} f(t)\,dt$.



Graph of f

(a) Find $g(0)$ and $g'(0)$.

The rubric requires this

$g(0) = \int_{-3}^{0} f(t)\,dt = \frac{1}{2}(3)(2+1) = \frac{9}{2}$ +1

$g'(x) = f(x) \cdot x'$

$g'(x) = f(x)$

$g'(0) = f(0)$

$g'(0) = 1$ +1

(b) Find all values of x in the open interval $(-5, 4)$ at which g attains a relative maximum. Justify your answer.

g attains a relative maximum at $x = 3$ b/c g' changes from positive to negative at $x = 3$ +1 +1

(c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.

At $x = -4$, $f(x)$ changes from negative to positive. +1

OPTIONAL

$g(-5) = \int_{-3}^{-5} f(t)\,dt = -0 = 0$

$g(-4) = \int_{-3}^{-4} f(t)\,dt = -1$ MIN

$g(4) = \int_{-3}^{4} f(t)\,dt = 9/2 + (2 - \frac{1}{2}\pi) + \frac{1}{2} - \frac{1}{2} = 6.5 - \frac{1}{2}\pi$

CANDIDATE'S TEST

x	g(x)
-5	0
-4	-1
4	$6.5 - \frac{1}{2}\pi$

+1

g has an absolute minimum value of -1 +1

(d) Find all values of x in the open interval $(-5, 4)$ at which the graph of g has a point of inflection.

(where does $g' = f$ change from inc. to dec. or dec. to inc.)

$x = -3, 1, 2$ +2