

# 2004 AP<sup>®</sup> CALCULUS AB

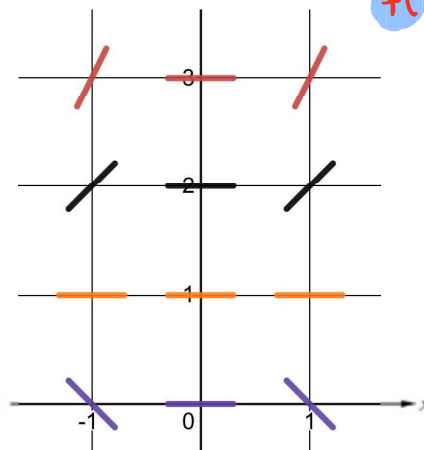
## Question 6

+1 Zero slopes  
+1 All other slopes

Consider the differential equation  $\frac{dy}{dx} = x^2(y-1)$ .

- (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.  
(Note: Use the axes provided in the pink test booklet.)

$(x, y)$	$x^2(y-1)$
$(\pm, 1)$	0
$(0, y)$	0
$(\pm, 3)$	2
$(\pm, 2)$	1
$(\pm, 0)$	



- (b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the  $xy$ -plane. Describe all points in the  $xy$ -plane for which the slopes are positive.

The slopes are positive in the  $xy$ -plane when  $x \neq 0$  and  $y > 1$ .

+1

- (c) Find the particular solution  $y = f(x)$  to the given differential equation with the initial condition  $f(0) = 3$ .

$$\frac{dy}{dx} = x^2(y-1)$$

$$\int \frac{1}{y-1} dy = \int x^2 dx \quad +1$$

$$+1 \ln|y-1| = \frac{1}{3}x^3 + C \quad +1$$

+1 at  $(0, 3)$   
 $\ln|3-1| = \frac{1}{3}(0)^3 + C$   
 $\ln|2| = C$   
 $\ln 2 = C \quad +1$

$$\ln|y-1| = \frac{1}{3}x^3 + \ln 2$$

$$|y-1| = e^{\frac{1}{3}x^3 + \ln 2}$$

$$y-1 = \pm e^{\frac{1}{3}x^3} \cdot e^{\ln 2}$$

$$y = 1 \pm 2e^{\frac{1}{3}x^3}$$

which EQ contains  $(0, 3)$

$$f(x) = 1 + 2e^{\frac{1}{3}x^3} \quad +1$$