

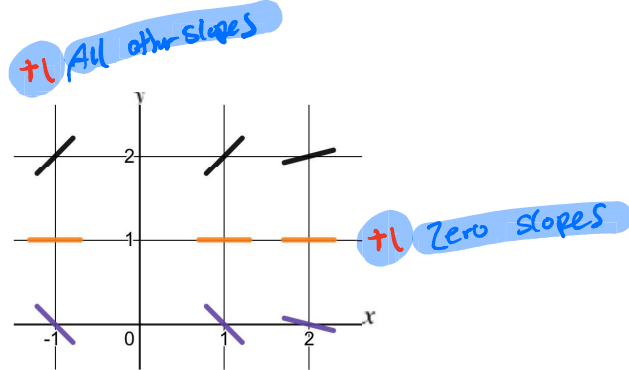
2008 AP<sup>®</sup> CALCULUS AB  
Question 5

Consider the differential equation  $\frac{dy}{dx} = \frac{y-1}{x^2}$ , where  $x \neq 0$ .

- (a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

(Note: Use the axes provided in the exam booklet.)

$(x, y)$	$\frac{y-1}{x^2}$
$(0, y)$	undefined
$(\pm 1, 0)$	-1
$(x, 1)$	0
$(\pm 1, 2)$	1
$(2, 0)$	$-\frac{1}{4}$
$(2, 2)$	$\frac{1}{4}$



- (b) Find the particular solution  $y = f(x)$  to the differential equation with the initial condition  $f(2) = 0$ .

$u = y - 1$   
 $du = dy$

$\int \frac{1}{y-1} dy = \int \frac{1}{x^2} dx$  +1

+1  $\ln|y-1| = -x^{-1} + C$  +1

+1 at  $(2, 0)$   
 $\ln|0-1| = -\frac{1}{2} + C$   
 $\ln|1| = -\frac{1}{2} + C$   
 $0 = -\frac{1}{2} + C$   
 $\frac{1}{2} = C$  +1

$\ln|y-1| = -\frac{1}{x} + \frac{1}{2}$

$|y-1| = e^{-\frac{1}{x} + \frac{1}{2}}$   
 $y-1 = \pm (e^{-\frac{1}{x} + \frac{1}{2}})$   
 $y = 1 \pm (e^{-\frac{1}{x} + \frac{1}{2}})$

Which one contains  $(2, 0)$ ?

+1  $f(x) = 1 - e^{-\frac{1}{x} + \frac{1}{2}}$

- (c) For the particular solution  $y = f(x)$  described in part (b), find

$\lim_{x \rightarrow \infty} f(x)$ .

$\lim_{x \rightarrow \infty} (1 - e^{-\frac{1}{x} + \frac{1}{2}}) = 1 - e^{0 + \frac{1}{2}} = 1 - e^{\frac{1}{2}}$  +1