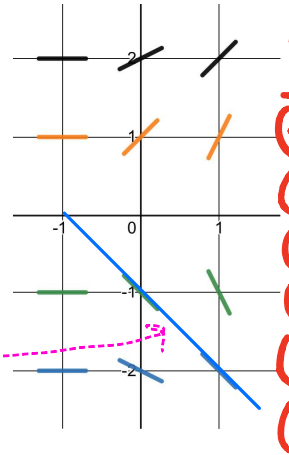
2010 AP® CALCULUS AB (Form B)
Question 5

Consider the differential equation $\frac{dy}{dx} = \frac{x+1}{y}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and for $-1 < x < 1$, sketch the solution curve that passes through the point $(0, -1)$.

(Note: Use the axes provided in the exam booklet.)



+1 Zero slopes
+1 All other slopes
+1 Solution curve

(x,y)	$\frac{x+1}{y}$
$(-1,y)$	0
$(0,2)$	$\frac{1}{2}$
$(0,1)$	1
$(0,-1)$	-1
$(0,-2)$	$-\frac{1}{2}$
$(1,2)$	1
$(1,1)$	2
$(1,-1)$	-2
$(1,-2)$	-1

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane for which $y \neq 0$. Describe all points in the xy-plane, $y \neq 0$, for which $\frac{dy}{dx} = -1$.

$\frac{x+1}{y} = -1$

$x+1 = -y$

$y = -x-1$

$\frac{dy}{dx} = -1$ for all points in the plane where $y = -x-1$ and $y \neq 0$. +1

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.

$\frac{dy}{dx} = \frac{x+1}{y}$

$\int y\,dy = \int (x+1)\,dx$ +1

$\frac{1}{2}y^2 = \frac{1}{2}x^2 + x + C$ +1

at $(0,3)$ +1

$\frac{1}{2}(3)^2 = \frac{1}{2}(0) + (0) + C$

$\frac{9}{2} = C$ +1

$\frac{1}{2}y^2 = \frac{1}{2}x^2 + x + \frac{9}{2}$

$y^2 = x^2 + 2x + 9$

$y = \pm\sqrt{x^2+2x+9}$

which one contains $(0,3)$

$f(x) = \sqrt{x^2+2x+9}$ +1