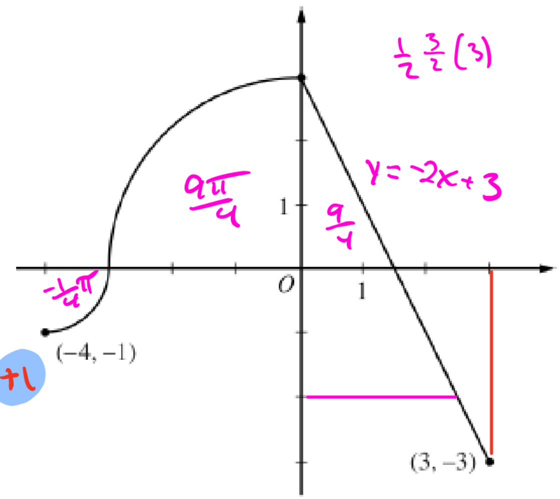
2011 AP® CALCULUS AB
Question 4

The continuous function f is defined on the interval $-4 \le x \le 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure above.



Graph of f

Let $g(x) = 2x + \int_0^x f(t)\,dt$.

(a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.

Area

$g(-3) = 2(-3) + \int_0^{-3} f(t)dt$

$= -6 - \int_{-3}^{0} f(t)dt$

$= -6 - \left(\frac{1}{4}\pi(3)^2\right)$

$= -6 - \left(\frac{1}{4}\pi 9\right)$

$g(-3) = -6 - \frac{9}{4}\pi$ +1

y-value

$g'(x) = 2 + f(x)\cdot x'$ +1

$g'(x) = 2 + f(x)$

$g'(-3) = 2 + f(-3)$

$= 2 + 0$

$g'(-3) = 2$ +1

(b) Determine the x-coordinate of the point at which g has an absolute maximum on the interval $-4 \le x \le 3$. Justify your answer.

EVT

CV

$g'(x) = 2 + f(x)$

$0 = 2 + f(x)$ +1

$f(x) = -2$

at $f(x) = -2$, $f(x) = -2x+3$

$-2 = -2x+3$

$-5 = -2x$

$\frac{5}{2} = x$ +1

EV $g(-4) = 2(-4) - \int_{-4}^{0} f(t)dt = -8 - \left(-\frac{\pi}{4} + \frac{9\pi}{4}\right) = -8 - 2\pi$

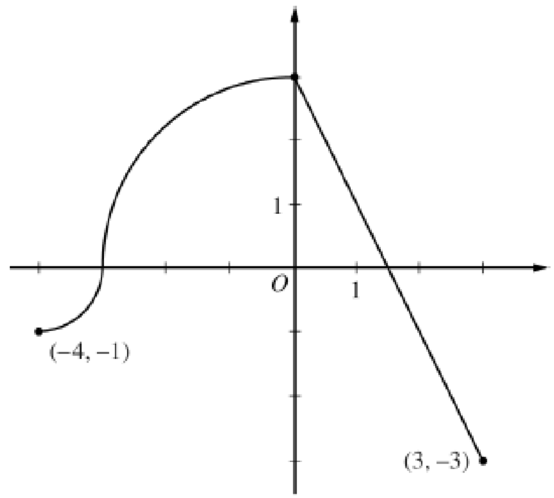
CV $g(5/2) = 2(5/2) + \int_0^{5/2} f(t)dt = 5 + \left(\frac{9}{4} - 1\right) = 5 + \frac{5}{4} = 6.25$

EV $g(3) = 2(3) + \int_0^3 f(t)dt = 6 + \left(\frac{9}{4} - \frac{1}{2}\cdot\frac{3}{2}(3)\right) = 6 + (0) = 6$

g has an absolute maximum at $x = 5/2$ +1

The continuous function f is defined on the interval $-4 \le x \le 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure above.

Let $g(x) = 2x + \int_0^x f(t)\,dt$.



Graph of f

(c) Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.

$g'(x) = 2 + f(x)$

$g''(x) = f'(x)$

$g''(x) > 0$ on $(-4, 0)$ and $g''(x) < 0$ on $(0, 3)$,

thus $g''(x) = f'(x)$ changes sign at $x = 0$.

$\therefore$ $g(x)$ has a point of inflection at $x = 0$.

+1

(d) Find the average rate of change of f on the interval $-4 \le x \le 3$. There is no point c, $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.

$$\text{A.R.C.} = \frac{f(3) - f(-4)}{3 - (-4)} = \frac{-3 - (-1)}{7}$$

$$\text{A.R.C.} = -\frac{2}{7}$$

+1

The M.V.T. cannot apply if $f(x)$ is not differentiable for each value on $(-4, 3)$.

$f(x)$ is not differentiable at $x = -3$ and $x = 0$.

+1