A GRAPHING CALCULATOR IS REQUIRED.

t (minutes)	0	1	5	6	8
$g(t)$ (cubic feet per minute)	12.8	15.1	20.5	18.3	22.7

9. Grain is being added to a silo. At time $t = 0$, the silo is empty. The rate at which the grain is being added is modeled by the differential function g, where $g(t)$ is measured in cubic feet per minute for $0 \leq t \leq 8$ minutes. Selected values of $g(t)$ are given in the table.

(a) Using the date in the table, approximate $g'(3)$. Using correct units, interpret the meaning of $g'(3)$ in context of the problem.

Part A

1 point is earned for: approximation

1 point is earned for: interpretation with units

$g'(3) \approx \frac{g(5)-g(1)}{5-1} = \frac{20.5-15.1}{4} = 1.35$

At time $t = 3$ minutes, the rate at which grain is being added to the silo is increasing at a rate of 1.35 cubic feet per minute per minute.

0	1	2

(b) Write an integral expression that represents the total amount of grain added to the silo from time $t = 0$ to time $t = 8$. Use at right Reman sum with the four subintervals indicated by the data in the table to approximate the interval

Part B

1 point is earned for: integral expression

1 point is earned for: right Riemann sum

1 point is earned for: approximation

The total amount of grain added to the silo from time $t = 0$ to time $t = 8$ is $\int_0^8 g(t)\,dt$ cubic feet.

$$\int_0^8 g(t)\,dt \approx g(1)\cdot(1-0) + g(5)\cdot(5-1) + g(6)\cdot(6-5) + g(8)\cdot(8-6)$$
$$= 15.1\cdot 1 + 20.5\cdot 4 + 18.3\cdot 1 + 22.7\cdot 2 = 160.8$$

0	1	2	3

(c) The grain int eh silo is spoiling at a rate modeled by $w(t) = 32\sqrt{\sin\left(\frac{\pi t}{74}\right)}$, where $w(t)$ is measured in cubic feet per minute for $0 \le t \le 8$ minutes. Using the result from part (b), approximate the amount of unsoiled grain remaining in the silo at time $t = 8$.

Part C

1 point is earned for: integral

1 point is earned for: answer

$$\int_0^8 w(t)\,dt = 99.051497$$

The approximate amount of unspoiled grain remaining in the silo at time $t = 8$ is $160.8 - \int_0^8 w(t)\,dt = 61.749$ (or 61.748) cubic feet.

0	1	2

(d) Based on the model in part (c), is the amount of unspoiled grain in the silo increasing or decreasing at time $t = 6$? Show the work that leads to your answer.

Part D

1 point is earned for: considers $g(6) - w(6)$

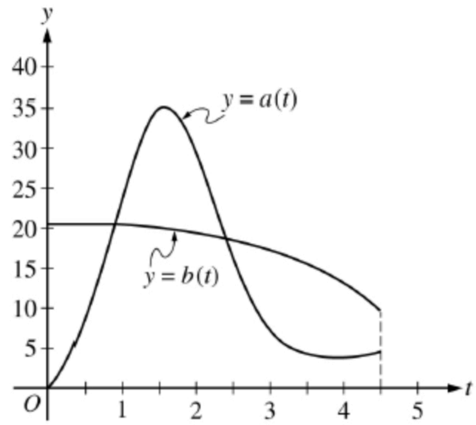
1 point is earned for: answer

$$g(6) - w(6) = 18.3 - 16.063173 = 2.236827 > 0$$

Because $g(6) - w(6) > 0$, the amount of unspoiled grain is increasing at time $t = 6$.

0	1	2

A GRAPHING CALCULATOR IS REQUIRED.



10. During the time interval $0 \leq t \leq 4.5$ hours, water flows into tank *A* at a rate of $a(t) = (2t - 5) + 5^{2\sin t}$ liters per hour. During the same time interval, water flows into tank *B* at a rate of $b(t)$ liters per hour. Both tanks are empty at time $t = 0$. The graphs of $y = a(t)$ and $y = b(t)$, shown in the figure above, intersect at $t = k$ and $t = 2.416$.

(a) How much water will be tank *A* at time $t = 4.5$?

Part A

1 point is earned for: integral

1 point is earned for: answer

$$\int_0^{4.5} a(t)\, dt = 66.532128$$

At time $t = 4.5$, tank *A* contains 66.532 liters of water.

0	1	2

(b) During the time interval $0 \leq t \leq k$ hours, water flows into tank *B* at a constant rate of 20.5 liters per hour. What is the difference between the amount of water in tank *A* and the amount of water in tank *B* at time $t = k$?

Part B

1 point is earned for: sets $a(k) = 20.5$

1 point is earned for: integral

1 point is earned for: answer

$$a(k) = 20.5 \Rightarrow k = 0.892040$$

$$\int_0^{k} (20.5 - a(t))\, dt = 10.599191$$

At time $t = k$, the difference in the amounts of water in the tanks is 10.599 liters.

0	1	2	3

(c) The area of the region bounded by the graphs of $y = a(t)$ and $y = b(t)$ for $k \leq t \leq 2.416$ is 14.470. How much water is in the tank *B* at time $t = 2.416$?

Part C

1 point is earned for: $\int_k^{2.416} a(t)\,dt$

1 point is earned for: answer

$$\int_0^{2.416} b(t)\,dt = \int_0^k b(t)\,dt + \int_k^{2.416} b(t)\,dt$$

$$\int_0^k b(t)\,dt = 20.5 \cdot k = 18.286826$$

On $k < t < 2.416$, tank *A* receives $\int_k^{2.416} a(t)\,dt = 44.497051$ liters of water, which is 14.470 more liters of water than tank *B*.

Therefore,

$$\int_k^{2.416} b(t)\,dt = \int_k^{2.416} a(t)\,dt - 14.470 = 30.027051.$$

$$\int_0^k b(t)\,dt + \int_k^{2.416} b(t)\,dt = 48.313876$$

At time $t = 2.416$, tank *B* contains 48.314 (or 48.313) liters of water.

0	1	2

(d) During the time interval $2.7 \leq t \leq 4.5$ hours, the rate at which water flows into tank *B* is modeled by $w(t) = 21 - \dfrac{30t}{(t-8)^2}$ liters per hour. Is the difference $w(t) - a(t)$ increasing or decreasing at time $t = 3.5$? Show the work that leads to your answer.

Part D

1 point is earned for: $w'(3.5) - a'(3.5) < 0$

1 point is earned for: conclusion

$$w'(3.5) - a'(3.5) = -1.14298 < 0$$

The difference $w(t) - a(t)$ is decreasing at $t = 3.5$.

0	1	2