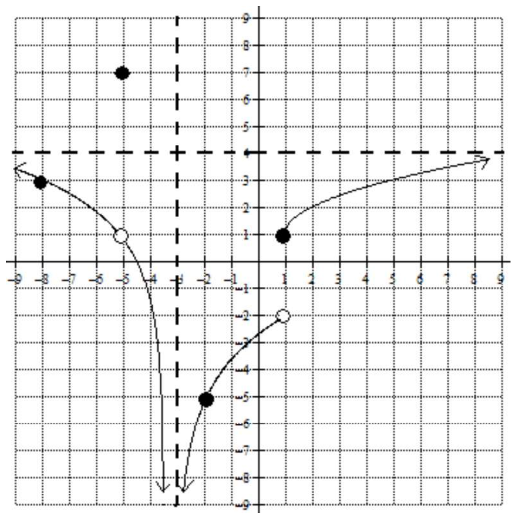
Topics 1.11 – 1.13

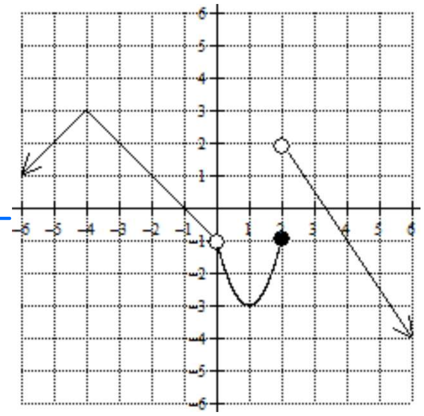
For exercises 1 – 3, determine if the function is continuous at each of the indicated values below. Use the three part definition of continuity to perform your analysis.

f(x)



1. $x = -5$	I. $f(-5) = 7$ $\therefore f(-5)$ is defined II. $\lim_{x\to -5^-} f(x) = 1 = \lim_{x\to -5^+} f(x)$ $\therefore \lim_{x\to -5} f(x) = 1$ & exists III $f(-5) \neq \lim_{x\to -5} f(x)$ $\therefore f(x)$ is not continuous at $x=-5$
2. $x = 1$	I. $f(1) = 1$ $\therefore f(1)$ is defined II $\lim_{x\to 1^-} f(x) = -2 \neq \lim_{x\to 1^+} f(x) = 1$ $\therefore \lim_{x\to 1} f(x)$ does not exist $\therefore f(x)$ is not continuous at $x=1$
3. $x = -2$	I. $f(-2) = -5$ $\therefore f(-2)$ is defined II $\lim_{x\to -2^-} f(x) = -5 = \lim_{x\to -2^+} f(x)$ $\therefore \lim_{x\to -2} f(x) = 5$ & exists III $f(-2) = \lim_{x\to -2} f(x) = 5$ $\therefore f(x)$ is continuous at $x=-2$

4. Use the three part definition of continuity to graphically justify why $p(x)$ is discontinuous at $x = 0$ and $x = 2$.



I. $p(x)$ is not defined at $x=0$ $\therefore$ $p(x)$ is discontinuous at $x=0$.

I $p(2)=-1$ $\therefore$ $p(2)$ is defined

II $\lim\limits_{x\to 2^-} p(x) = -1 \neq \lim\limits_{x\to 2^+} p(x) = 2$

$\therefore \lim\limits_{x\to 2} p(x)$ does not exist

$\therefore$ $p(x)$ is discontinuous at $x=2$.

5. For what values of k and m is the function $g(x)$ everywhere continuous? Use limits to set up your work.

$$g(x)=\begin{cases} kx^2+m, & x<-1 \\ e^{\ln(2x+3)}, & -1\le x\le 3 \quad \to 2x+3 \\ kx-m, & x>3 \end{cases}$$

$\lim\limits_{x\to -1^-} g(x) = \lim\limits_{x\to -1^+} g(x)$

$\lim\limits_{x\to -1^-}(kx^2+m) = \lim\limits_{x\to -1^+}(2x+3)$

$k(-1)^2+m = 2(-1)+3$

$k+m=-2+3$

$k+m=1$

$\lim\limits_{x\to 3^-} g(x) = \lim\limits_{x\to 3^+} g(x)$

$\lim\limits_{x\to 3^-}(2x+3) = \lim\limits_{x\to 3^+}(kx-m)$

$2(3)+3 = k(3)-m$

$9 = 3k-m$

$1 = k+m$

$9 = 3k-m$

$10 = 4k$

$\frac{10}{4} = k$

$\frac{5}{2} = k$

$1 = k+m$

$\frac{2}{2} = \frac{5}{2}+m$

$-\frac{3}{2} = m$

Find the value of a that makes each of the functions below everywhere continuous. Write the two limits that must be equal in order for the function to be continuous.

6. $f(x)=\begin{cases} 4-x^2, & x<-1 \\ ax^2-1, & x\ge -1 \end{cases}$

$\lim\limits_{x\to -1^-}(4-x^2) = \lim\limits_{x\to -1^+}(ax^2-1)$

$4-(-1)^2 = a(-1)^2-1$

$4-1 = a-1$

$3 = a-1$

$4 = a$

7. $f(x)=\begin{cases} x^2+x+a, & x<2 \\ ax^3-x^2, & x\ge 2 \end{cases}$

$\lim\limits_{x\to 2^-}(x^2+x+a) = \lim\limits_{x\to 2^+}(ax^3-x^2)$

$(2)^2+(2)+a = a(2)^3-(2)^2$

$4+2+a = 8a-4$

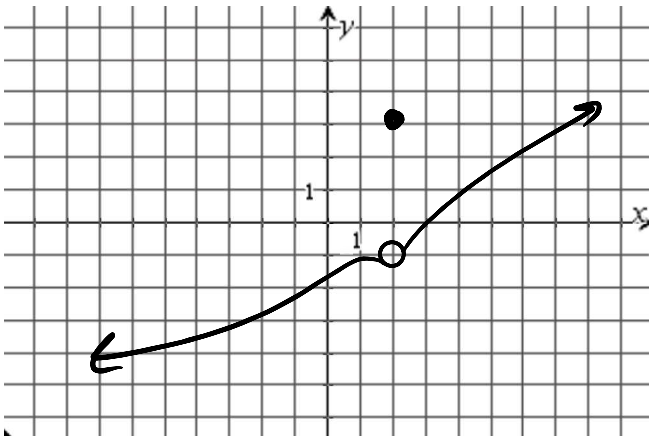
$6+a = 8a-4$

$10 = 7a$

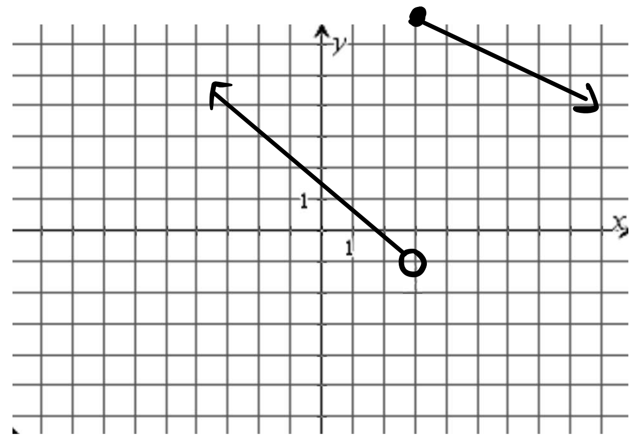
$10/7 = a$

8) Sketch a function having the following attributes. Graphs will vary.

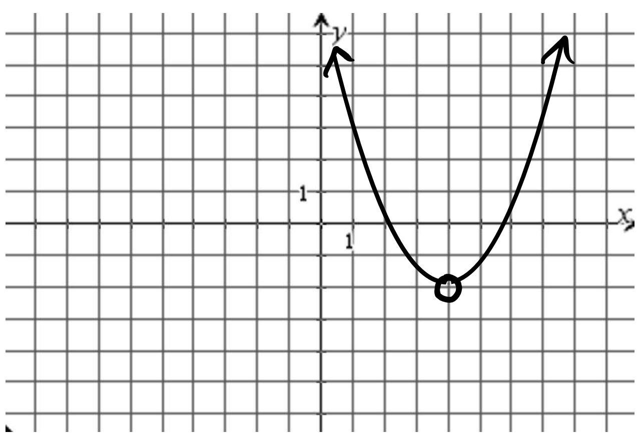
a.) has a value of $f(2)$, a limit as x approaches 2, but is not continuous at $x = 2$.



b.) has a "step" or "jump" discontinuity at $x = 3$ where $f(3) = 7$.



c.) $\lim_{x \to 4} f(x) = -2$ but the function is not continuous at $x = 4$.



d.) the value of $f(-2) = 3$ but there is no limit of $f(x)$ as x approaches -2 and no vertical asymptotes there.