Topic 1.14 – Connecting Infinite Limits and Vertical Asymptotes

Find each of the following limits.

1. $\lim_{x\to 0^-} \frac{1}{x^2}$ $= \infty$, dne

$x \to 0^-$	$\frac{1}{x^2}$
$-.1$	$\frac{+}{+} = +$

2. $\lim_{x\to 2^-} \frac{x+3}{x^2+x-6}$ $= \lim_{x\to 2^-} \frac{x+3}{(x+3)(x-2)}$

$x \to 2^-$	$\frac{1}{x-2}$
-2.1	$\frac{1}{-2.1-2} = \frac{+}{-} = \text{NEG}$

$= \lim_{x\to 2^-} \frac{1}{x-2}$

$= -\infty$, dne

3. $\lim_{x\to 3^-} \frac{x}{x-3}$ $= -\infty$, dne

$x \to 3^-$	$\frac{x}{x-3}$
2.9	$\frac{2.9}{2.9-3} = \frac{+}{-} = \text{neg}$

4. $\lim_{x\to 5^+} \frac{x}{x^2-25}$ $= \infty$, dne

$x \to 5^+$	$\frac{x}{(x-5)(x+5)}$
5.1	$\frac{5.1}{(5.1-5)(5+5)} = \frac{+}{(+)(+)} = \text{pos}$

5. $\lim_{y\to 6^+} \frac{y+6}{y^2-6}$ $= \infty$, dne

6. $\lim_{x\to 4} \frac{3-x}{x^2-2x-8}$ $= \lim_{x\to 4} \frac{3-x}{(x-4)(x+2)}$ dne

$\lim_{x\to 4^-} \frac{3-x}{(x-4)(x+2)} = \frac{\text{neg}}{(\text{neg})(\text{pos})} = \text{pos}$

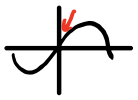
$\lim_{x\to 4^+} \frac{3-x}{(x-4)(x+2)} = \frac{\text{neg}}{(\text{pos})(\text{pos})} = \text{NEG}$

7. $\lim_{x\to 5} \frac{x}{|x-5|}$ dne

$\lim_{x\to 5^-} \frac{x}{|x-5|} = \frac{\text{neg}}{\text{pos}} = -\infty$

$\lim_{x\to 5^+} \frac{x}{|x-5|} = \frac{\text{pos}}{\text{pos}} = \infty$

8. $\lim_{x\to 0} \frac{2}{\sin(x)}$ dne



$\lim_{x\to 0^+} \frac{2}{\sin(x)} = \infty$

$\lim_{x\to 0^-} \frac{2}{\sin(x)} = -\infty$

9. $\lim_{x\to -4} \left(x^2 + \frac{2}{x+4}\right)$ dne

$\lim_{x\to -4^-} \left(x^2 + \frac{2}{x+4}\right) = 16 + \frac{2}{\text{neg}} = -\infty$

$\lim_{x\to -4^+} \left(x^2 + \frac{2}{x+4}\right) = 16 + \frac{2}{\text{pos}} = \infty$

Determine the equation(s) of the vertical asymptote for the graph of each function.

10. $f(x)=\dfrac{x^2-2x-15}{x^3-5x^2+x-5}$ $=\dfrac{(x-5)(x+3)}{(x-5)(x^2+1)}$

hole

$x^2(x-5)$

VA when $x^2+1=0$

$x^2=-1$

$x=\pm\sqrt{-1}$

No real solution

$\therefore$ No VA.

11. $f(x)=\dfrac{1}{e^x-1}$

VA when $e^x-1=0$

$e^x=1$

$x=0$

$\therefore$ VA @ $x=0$

12. $f(x)=\dfrac{x}{\sin x}$

VA when $\sin(x)=0$

$\therefore x=\ldots,-2\pi,-\pi,0,\pi,2\pi,\ldots$

$\therefore$ VA @ $x=k\cdot\pi,\ k\in\mathbb{Z}$