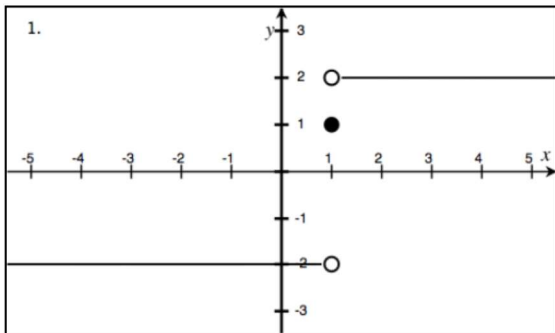
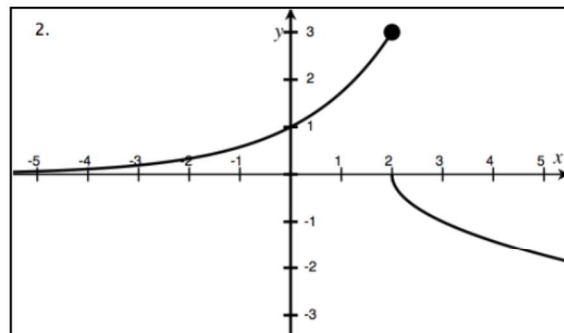


Skill Builder: Topic 1.15 – Connecting Limits at Infinity and Horizontal Asymptotes

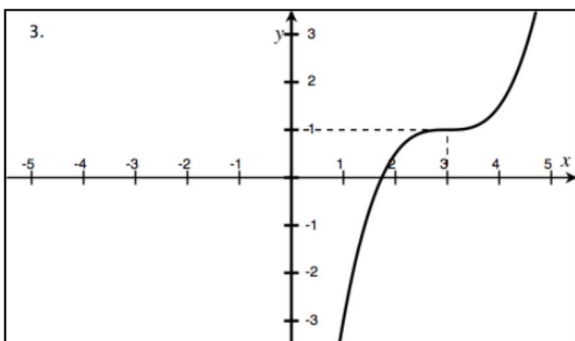
Find each limit as x approaches ∞ or $-\infty$.



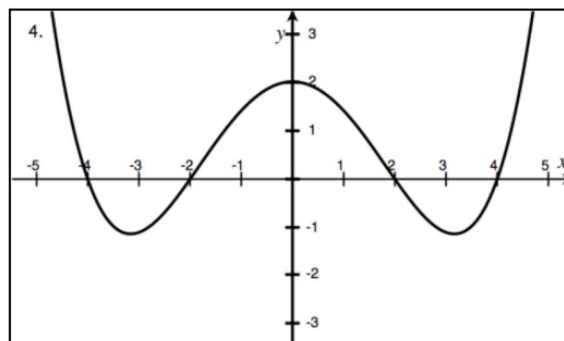
a.) $\lim_{x \rightarrow -\infty} f(x) = -2$	b.) $\lim_{x \rightarrow \infty} f(x) = 2$
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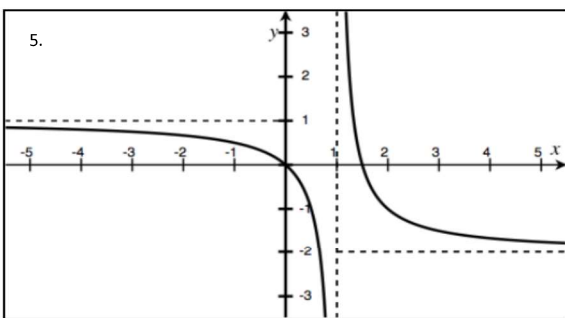
a.) $\lim_{x \rightarrow -\infty} f(x) = 0$	b.) $\lim_{x \rightarrow \infty} f(x) = -\infty$
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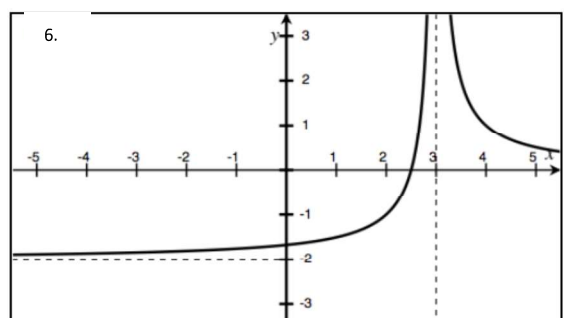
a.) $\lim_{x \rightarrow -\infty} f(x) = -\infty$	b.) $\lim_{x \rightarrow \infty} f(x) = \infty$
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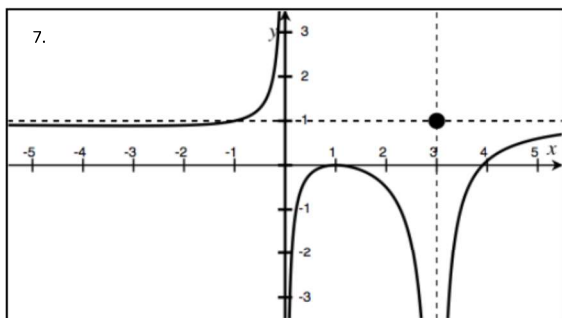
a.) $\lim_{x \rightarrow -\infty} f(x) = \infty$	b.) $\lim_{x \rightarrow \infty} f(x) = \infty$
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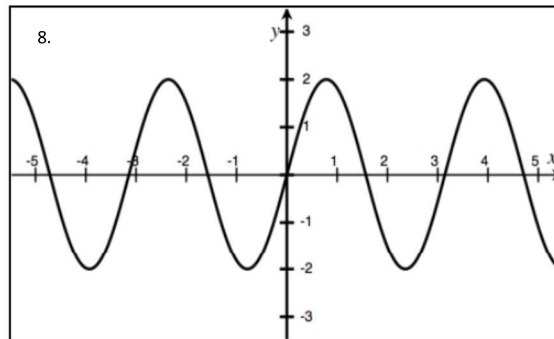
a.) $\lim_{x \rightarrow -\infty} f(x) = 1$	b.) $\lim_{x \rightarrow \infty} f(x) = -2$
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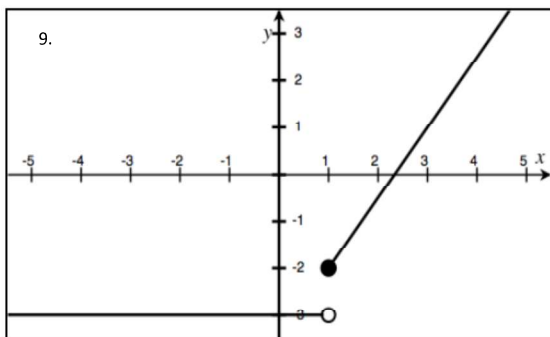
a.) $\lim_{x \rightarrow -\infty} f(x) = -2$	b.) $\lim_{x \rightarrow \infty} f(x) = 0$
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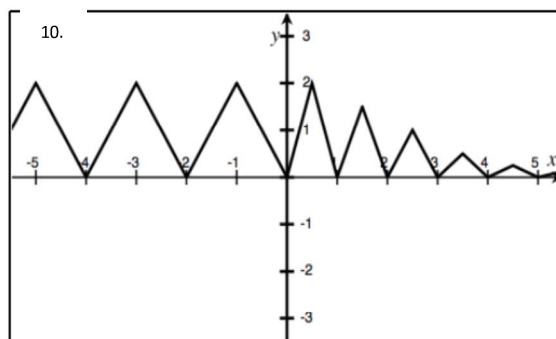
a.) $\lim_{x \rightarrow -\infty} f(x) = 1$ b.) $\lim_{x \rightarrow \infty} f(x) = 1$



a.) $\lim_{x \rightarrow -\infty} f(x)$ dne b.) $\lim_{x \rightarrow \infty} f(x)$ dne



a.) $\lim_{x \rightarrow -\infty} f(x) = -3$ b.) $\lim_{x \rightarrow \infty} f(x) = \infty$



a.) $\lim_{x \rightarrow -\infty} f(x)$ dne b.) $\lim_{x \rightarrow \infty} f(x) = 0$

11.) $\lim_{x \rightarrow \infty} 6 = 6$

12.) $\lim_{x \rightarrow \infty} (-2x + 11) = -\infty$

13.) $\lim_{x \rightarrow -\infty} (3x^4 - 3x^3 + 5x^2 + 8x - 3) = \infty$

14.) $\lim_{x \rightarrow \infty} \frac{2x-3}{4x+5} = \frac{1}{2}$ (Same)

15.) $\lim_{x \rightarrow -\infty} \frac{7-3x^3}{2x^3+1} = \frac{1}{2}$ (Same)

16.) $\lim_{x \rightarrow \infty} \frac{2}{5x-3} = 0$ (faster)

17.) $\lim_{x \rightarrow -\infty} \frac{2x+30}{6x^{12}-5} = 0$ (faster)

18.) $\lim_{x \rightarrow \infty} \frac{4x^4}{6x^3-19} = \infty$ (faster)

19.) $\lim_{x \rightarrow -\infty} \frac{4x^2-3x-2}{9x^2+9x+7} = \infty$ (faster)

$$20.) \lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2+4}} = 1$$

$$= \lim_{x \rightarrow \infty} \frac{x^+}{|x|^+} = 1$$

$$21.) \lim_{x \rightarrow -\infty} \frac{x}{\sqrt{x^2+4}} = -1$$

$$= \lim_{x \rightarrow -\infty} \frac{x^-}{|x|^+} = -1$$

$$22.) \lim_{x \rightarrow -\infty} \frac{\sqrt{3x^2+x}}{x^2-1} = 0$$

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{3}|x|}{x^2} = 0$$

FASTER

Determine the equation(s) of each horizontal asymptote. Show your limits.

$$23.) f(x) = \frac{2x^2 - 3x + 5}{x^2 + 1}$$

Same

HA @ $y = 2$

$$24.) f(x) = \frac{x}{x^2 - 1}$$

FASTER

HA @ $y = 0$

$$25.) f(x) = \frac{\sqrt{x^2} - 1}{x^3 - 1}$$

FASTER

HA @ $y = 0$

$$26.) f(x) = \frac{2x+1}{\sqrt{x^2} - x}$$

Same

$$\lim_{x \rightarrow -\infty} \frac{2x}{|x|} = -2 \quad \therefore \text{HA @ } y = -2$$

$$\lim_{x \rightarrow \infty} \frac{2x}{|x|} = 2 \quad \therefore \text{HA @ } y = 2$$

Determine the equation of the slant asymptote.

$$27.) f(x) = \frac{x^3}{x^2 - 9}$$

$$\begin{array}{r} x \\ x^2 + 0x - 9 \overline{) x^3 + 0x^2 + 0x + 0} \\ \underline{-x^3 + 0x^2 + 9x} \\ 9x \end{array}$$

$$\text{SA @ } y = x$$

$$28.) f(x) = \frac{x^2 - 6x + 12}{x - 4}$$

$$\begin{array}{r} x - 2 \\ x - 4 \overline{) x^2 - 6x + 12} \\ \underline{-x^2 + 4x} \\ -2x + 12 \\ \underline{+2x - 8} \\ 4 \end{array}$$

$$\text{SA @ } y = x - 2$$