Skill Builder: Topic 1.16 – Working with the Intermediate Value Theorem

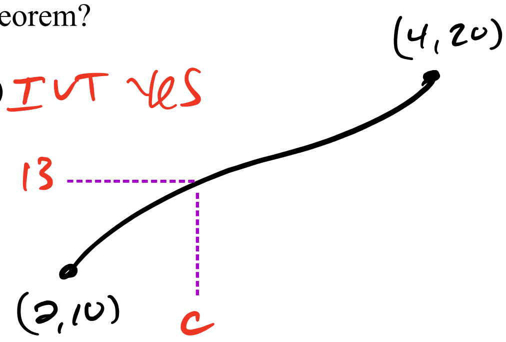
1. Let f be a function that is continuous on the closed interval $[2,4]$ with $f(2)=10$ and $f(4)=20$. Which of the following is guaranteed by the Intermediate Value Theorem?

(A) $f(x)=13$ has at least one solution in the open interval $(2,4)$ IVT yes

(B) $f(3)=15$ maybe

(C) f attains a maximum on the open interval $(2,4)$ maybe

(D) $f(x)=10$ at some other value(s) of x other than $x=2$ maybe



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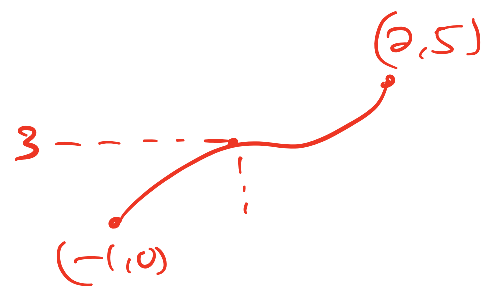
2. Let g be a function such that $g(-1)=0$ and $g(2)=5$. Which of the following conditions guarantees that there is an x, $-1<x<2$, for which $g(x)=3$?

(A) g is defined for all x in $(-1,2)$

(B) g is continuous for all x in $[-1,2]$

(C) g is increasing on $[-1,2]$

(D) there exists an x in $(-1,2)$ such that $g(x)=6$



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x	0	4	6	8	13
$f(x)$	3	4.5	3	2.5	4.4

4 4 4

3. The table above shows selected values of a continuous function f. For $0 \le x \le 13$, what is the fewest possible number of times that $f(x)=4$?

(A) One (B) Two (C) Three (D) Four

4. Determine, using the intermediate value theorem, if the function $F(x) = x^3 + 2x - 1$ has a zero on the interval [0, 1]. Justify your answer and find the indicated zero, if it exists.

I $F(x)$ is continuous on $[0,1]$

II. $F(0) = 0^3 + 2(0) - 1 = -1$
$F(1) = (1)^3 + 2(1) - 1 = 2$ } $F(0) < F(c) = 0 < 2$

∴ The IVT guarentees a value of c on $(0,1)$ such that $F(c) = 0$.

$0 = c^3 + 2c - 1$

$c \approx 0.453$

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5. Determine, using the intermediate value theorem, if the function $g(\vartheta) = \vartheta^2 - 2 - \cos\vartheta$ has a zero on the interval [0, π]. Justify your answer and find the indicated zero, if it exists.

I $g(\theta)$ is continuous on $[0,\pi]$

II. $g(0) = 0^2 - 2 - \cos(0) = -2 - 1 = -3$
$g(\pi) = \pi^2 - 2 - \cos(\pi) = \pi^2 - 2 + 1 = \pi^2 - 1 \approx 8.870$ } $g(0) < g(c) = 0 < g(\pi)$

∴ The IVT guarantees a value of c on $(0,\pi)$ such that $g(c) = 0$

$0 = c^2 - 2 - \cos(c)$

$c \approx 1.455$

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6. First, verify that the I.V.T. is applicable for the given function on the given interval. Then, if it is applicable, find the value of the indicated c, guaranteed by the theorem.

$f(x) = x^2 - 6x + 8$ Interval: [0, 3] $f(c) = 0$

I $f(x)$ is continuous on $[0, 3]$

II. $f(0) = 8$
$f(3) = 3^2 - 6(3) + 8 = 9 - 18 + 8 = -1$
$f(3) < f(c) = 0 < f(0)$

∴ The IVT guarantees a value of c on $(0,3)$ such that $f(c) = 0$.

$c^2 - 6c + 8 = 0$
$(c-4)(c-2) = 0$
$\cancel{c=4}$, $c = 2$

x	$f(x)$	$g(x)$
1	6	2
2	9	3
3	10	4
6	−1	6
10	3	11

7. The functions f and g are continuous for all real numbers, and g is strictly increasing. The table above gives values of the functions at selected values of x. The function h is given by $h(x) = g(f(x))$.

Explain why there must be a value r for $1 < r < 3$ such that $h(r) = 8$.

1) h is continuous

2) $h(1) = g(f(1)) = g(6) = -1$; $h(3) = g(f(3)) = g(10) = 11$ $\Big\}$ $h(1) < 8 < h(3)$

∴ IVT guarantees a value of r on (1,3) such that h(r) = 8.

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t (minutes)	0	2	5	8	12
$v_A(t)$ (meters / minute)	0	100	40	− 120	− 150

8. Train A runs back and forth on an east-west section of railroad track. Train A's velocity, measured in meters per minute, is given by a continuous function $v_A(t)$, where t is measured in minutes. Selected values for $v_A(t)$ are given in the table above.

Do the data in the table support the conclusion that Train A's velocity is -100 meters per minute at some time t with $5 < t < 8$? Give a reason for your answer.

1) $V_A(t)$ is continuous

2) $v(12) < -100 < v(5)$

∴ IVT guarantees a value of c such that v(c) = −100 m/min