

Homework 1.3

Find the value of each limit. For a limit that does not exist, state why.

1. $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ if $f(x) = 3x^2 - 2x$

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{3(x+h)^2 - 2(x+h) - (3x^2 - 2x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{3x^2 + 6hx + 3h^2 - 2x - 2h - 3x^2 + 2x}{h} \\ &= \lim_{h \rightarrow 0} \frac{6hx + 3h^2 - 2h}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(6x + 3h - 2)}{h} \\ &= \lim_{h \rightarrow 0} (6x + 3h - 2) = 6x + 3(0) - 2 \\ &= 6x - 2 \end{aligned}$$

2. $\lim_{x \rightarrow 2} \frac{h(x) - h(0)}{x}$ if $h(x) = x^2 + 2x - 3$

$$\begin{aligned} &= \lim_{x \rightarrow 2} \frac{x^2 + 2x - 3 - (0^2 + 2(0) - 3)}{x} \\ &= \lim_{x \rightarrow 2} \frac{x^2 + 2x - 3 + 3}{x} \\ &= \lim_{x \rightarrow 2} \frac{x^2 + 2x}{x} \\ &= \lim_{x \rightarrow 2} \frac{x(x+2)}{x} \\ &= \lim_{x \rightarrow 2} (x+2) = 2+2 = 4 \end{aligned}$$

If $\lim_{x \rightarrow 3} f(x) = 2$ and $\lim_{x \rightarrow 3} g(x) = -4$, find each of the following limits. Show your analysis applying the limit properties.

3. $\lim_{x \rightarrow 3} \left[\frac{5f(x)}{g(x)} \right] = 5 \cdot \frac{\lim_{x \rightarrow 3} f(x)}{\lim_{x \rightarrow 3} g(x)}$

$$\begin{aligned} &= 5 \cdot \frac{2}{-4} \\ &= 5 \left(\frac{1}{-2} \right) \\ &= \frac{5}{-2} \end{aligned}$$

4. $\lim_{x \rightarrow 3} [f(x) + 2g(x)]$

$$\begin{aligned} &= \lim_{x \rightarrow 3} f(x) + 2 \lim_{x \rightarrow 3} g(x) \\ &= 2 + 2(-4) \\ &= 2 - 8 \\ &= -6 \end{aligned}$$

5. $\lim_{x \rightarrow 3} \sqrt{4f(x)} = 2 \cdot \sqrt{\lim_{x \rightarrow 3} f(x)}$

$$\begin{aligned} &= 2 \cdot \sqrt{2} \\ &= 2\sqrt{2} \end{aligned}$$

6. $\lim_{x \rightarrow 3} \frac{g(x)}{8} = \frac{1}{8} \lim_{x \rightarrow 3} g(x)$

$$\begin{aligned} &= \frac{1}{8} (-4) \\ &= -\frac{1}{2} \end{aligned}$$

7. $\lim_{x \rightarrow 3} [3f(x) - g(x)]$

$$\begin{aligned} &= 3 \lim_{x \rightarrow 3} f(x) - \lim_{x \rightarrow 3} g(x) \\ &= 3(2) - (-4) \\ &= 6 + 4 \\ &= 10 \end{aligned}$$

8. $\lim_{x \rightarrow 3} \left[\frac{f(x)g(x)}{12} \right]$

$$\begin{aligned} &= \frac{1}{12} \left(\lim_{x \rightarrow 3} f(x) \right) \left(\lim_{x \rightarrow 3} g(x) \right) \\ &= \frac{1}{12} (2)(-4) \\ &= -\frac{8}{12} \\ &= -\frac{2}{3} \end{aligned}$$

If $\lim_{x \rightarrow 4} f(x) = 0$ and $\lim_{x \rightarrow 4} g(x) = 3$, find each of the following limits. Show your analysis applying the properties of limits.

9. $\lim_{x \rightarrow 4} \left[\frac{g(x)}{f(x)-1} \right] = \lim_{x \rightarrow 4} \frac{g(x)}{f(x)-1}$

$$\begin{aligned} &= \frac{\lim_{x \rightarrow 4} g(x)}{\lim_{x \rightarrow 4} f(x) - 1} \\ &= \frac{3}{0-1} \\ &= \frac{3}{-1} \\ &= -3 \end{aligned}$$

10. $\lim_{x \rightarrow 4} (xf(x))$

$$\begin{aligned} &= \lim_{x \rightarrow 4} x \cdot \lim_{x \rightarrow 4} f(x) \\ &= 4 \cdot 0 \\ &= \underline{\underline{0}} \end{aligned}$$

11. $\lim_{x \rightarrow 4} [g(x) + 3]$

$$\begin{aligned} &= \lim_{x \rightarrow 4} g(x) + \lim_{x \rightarrow 4} 3 \\ &= 3 + 3 \\ &= 6 \end{aligned}$$

12. $\lim_{x \rightarrow 4} g^2(x) = \left[\lim_{x \rightarrow 4} g(x) \right]^2$

$$\begin{aligned} &= (3)^2 \\ &= 9 \end{aligned}$$