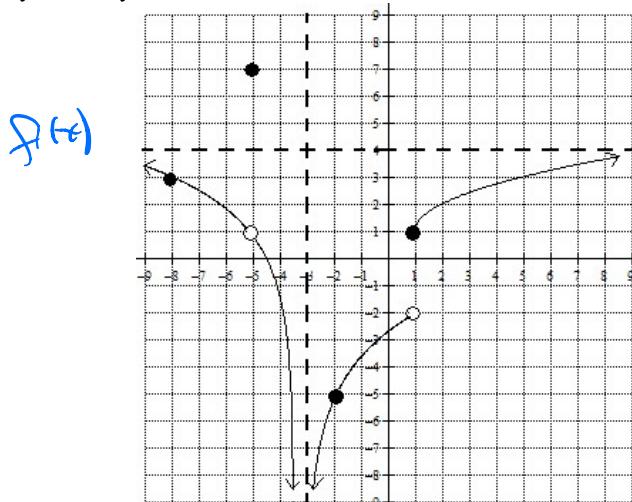


Homework 1.6

For exercises 1 – 3, determine if the function is continuous at each of the indicated values below. Use the three part definition of continuity to perform your analysis.



1. $x = -5$

I. $f(-5) = 1$

 $\therefore f(-5)$ is defined

II. $\lim_{x \rightarrow -5} f(x) = 1 = \lim_{x \rightarrow -5^+} f(x)$

 $\therefore \lim_{x \rightarrow -5} f(x) = 1 \notin \text{exists}$

III. $f(-5) \neq \lim_{x \rightarrow -5} f(x)$

 $\therefore f(x)$ is not continuous at $x = -5$

2. $x = 1$

I. $f(1) = 1 \therefore f(1)$ is defined

II. $\lim_{x \rightarrow 1^-} f(x) = -2 \neq \lim_{x \rightarrow 1^+} f(x) = 1$

 $\therefore \lim_{x \rightarrow 1} f(x)$ does not exist $\therefore f(x)$ is not continuous at $x = 1$

3. $x = -2$

I. $f(-2) = -5 \therefore f(-2)$ is defined

II. $\lim_{x \rightarrow -2^-} f(x) = -5 = \lim_{x \rightarrow -2^+} f(x)$

 $\therefore \lim_{x \rightarrow -2} f(x) = 5 \notin \text{exists}$

III. $f(-2) = \lim_{x \rightarrow -2} f(x) = 5$

 $\therefore f(x)$ is continuous at $x = -2$ 

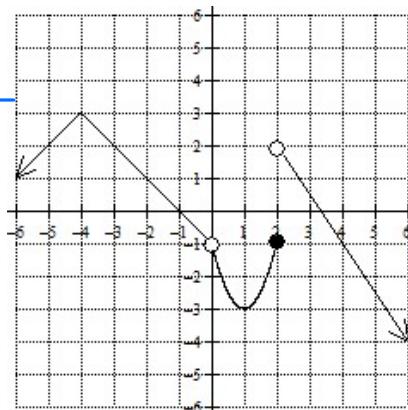
4. Use the three part definition of continuity to graphically justify why $p(x)$ is discontinuous at $x = 0$ and $x = 2$.

I. $p(x)$ is not defined at $x=0 \therefore p(x)$ is discontinuous at $x=0$.

II. $\lim_{x \rightarrow 2^-} p(x) = -1 \neq \lim_{x \rightarrow 2^+} p(x) = 2$

$\therefore \lim_{x \rightarrow 2} p(x)$ does not exist

$\therefore p(x)$ is discontinuous at $x=2$.



5. For what values of k and m is the function $g(x)$ everywhere continuous? Use limits to set up your work.

$$g(x) = \begin{cases} kx^2 + m, & x < -1 \\ e^{\ln(2x+3)}, & -1 \leq x \leq 3 \\ kx - m, & x > 3 \end{cases}$$

$$\lim_{x \rightarrow -1^-} g(x) = \lim_{x \rightarrow -1^+} g(x)$$

$$\lim_{x \rightarrow -1^-} g(x) = \lim_{x \rightarrow -1^+} g(x)$$

$$l = k + m$$

$$\lim_{x \rightarrow -1^-} (kx^2 + m) = \lim_{x \rightarrow -1^+} (2x + 3)$$

$$\lim_{x \rightarrow -1^-} (2x + 3) = \lim_{x \rightarrow -1^+} (kx - m)$$

$$9 = 3k - m$$

$$k(-1)^2 + m = 2(-1) + 3$$

$$2(-1)^2 + m = 2(3) - m$$

$$10 = 4k$$

$$k + m = -2 + 3$$

$$9 = 3k - m$$

$$\frac{10}{4} = k$$

$$k + m = 1$$

$$\frac{5}{2} = k$$

$$\lim_{x \rightarrow -1^-} (4-x^2) = \lim_{x \rightarrow -1^+} (ax^2 - 1)$$

$$l = k + m$$

$$4 - (-1)^2 = a(-1)^2 - 1$$

$$\frac{2}{2} = \frac{5}{2} + m$$

$$4 - 1 = a - 1$$

$$-\frac{3}{2} = m$$

$$3 = a - 1$$

$$\frac{5}{2} = \frac{5}{2} + m$$

$$4 = a$$

$$6 + a = 8a - 4$$

$$6 + a = 8a - 4$$

$$10 = 7a$$

$$10/7 = a$$

Find the value of a that makes each of the functions below everywhere continuous. Write the two limits that must be equal in order for the function to be continuous.

6. $f(x) = \begin{cases} 4 - x^2, & x < -1 \\ ax^2 - 1, & x \geq -1 \end{cases}$

7. $f(x) = \begin{cases} x^2 + x + a, & x < 2 \\ ax^3 - x^2, & x \geq 2 \end{cases}$

$$\lim_{x \rightarrow 2^-} (x^2 + x + a) = \lim_{x \rightarrow 2^+} (ax^3 - x^2)$$

$$(2)^2 + (2) + a = a(2)^3 - (2)^2$$

$$4 + 2 + a = 8a - 4$$

$$6 + a = 8a - 4$$

$$10 = 7a$$

$$10/7 = a$$