Homework 1.7

1. Determine, using the intermediate value theorem, if the function $F(x) = x^3 + 2x - 1$ has a zero on the interval [0, 1]. Justify your answer and find the indicated zero, if it exists.

I $F(x)$ is continuous on $[0,1]$

II. $F(0) = 0^3 + 2(0) - 1 = -1$

$F(1) = (1)^3 + 2(1) - 1 = 2$

$F(0) < F(c) = 0 < 2$

$\therefore$ The IVT guarentees a value of c on $(0,1)$ such that $F(c) = 0$.

$0 = c^3 + 2c - 1$

$c \approx 0.453$

2. Determine, using the intermediate value theorem, if the function $g(\theta) = \theta^2 - 2 - \cos\theta$ has a zero on the interval $[0, \pi]$. Justify your answer and find the indicated zero, if it exists.

I $g(\theta)$ is continuous on $[0, \pi]$

II. $g(0) = 0^2 - 2 - \cos(0) = -2 - 1 = -3$

$g(\pi) = \pi^2 - 2 - \cos(\pi) = \pi^2 - 2 + 1 = \pi^2 - 1 \approx 8.870$

$g(0) < g(c) = 0 < g(\pi)$

$\therefore$ The IVT guarantees a value of c on $(0, \pi)$ such that $g(c) = 0$

$0 = c^2 - 2 - \cos(c)$

$c \approx 1.455$

For exercises 3 – 5, first, verify that the I.V.T. is applicable for the given function on the given interval. Then, if it is applicable, find the value of the indicated c, guaranteed by the theorem.

3. $f(x) = x^2 - 6x + 8$ Interval: [0, 3] $f(c) = 0$

I $f(x)$ is continuous on $[0, 3]$

II. $f(0) = 8$

$f(3) = 3^2 - 6(3) + 8 = 9 - 18 + 8 = -1$

$f(3) < f(c) = 0 < f(0)$

$\therefore$ The IVT guarantees a value of c on $(0,3)$ such that $f(c) = 0$.

$c^2 - 6c + 8 = 0$

$(c-4)(c-2) = 0$

$\cancel{c=4}$, $c = 2$

4. $g(x) = x^3 - x^2 + x - 2$ Interval: $[0, 3]$ $g(c) = 4$

I. $g(x)$ is continuous on $[0,3]$

II. $g(0) = -2$

$g(3) = 19$

$g(0) < g(c) = 4 < g(3)$

$\therefore$ The IVT guarantees a value of c on $(0,3)$ such that $f(c) = 4$

$4 = c^3 - c^2 + c - 2$

$0 = c^3 - c^2 + c - 6$

$c = 2$

5. $h(x) = \frac{x^2+x}{x-1}$ Interval: $\left[\frac{5}{2}, 4\right]$ $h(c) = 6$

I. $h(x)$ is continuous on $[5/2, 4]$

II $h(5/2) = \dfrac{(5/2)^2 + 5/2}{5/2 - 1} = \dfrac{\frac{25}{4} + \frac{10}{4}}{\frac{10}{4} - \frac{4}{4}} = \dfrac{\frac{35}{\cancel{4}} \cdot 4}{\frac{6}{\cancel{4}} \cdot 4} = 35/6$

$h(4) = \dfrac{4^2 + 4}{4 - 1} = \dfrac{16 + 4}{3} = \dfrac{20}{3}$

$h(5/2) < h(c) = 6 < h(4)$

$\therefore$ The IVT guarantees a value of c on $(5/2, 4)$ such that $f(c) = 6$.

$\dfrac{c^2 + c}{c - 1} = 6$

$c^2 + c = 6c - 6$

$c^2 - 5c + 6 = 0$

$(c - 3)(c - 2) = 0$

$c = 3$ $\quad c \neq 2$