

Homework 1.8

For exercises 1 – 3, find the limit indicated. Explain what the result of the limit means about the graph of the given rational function.

<p>1. $\lim_{x \rightarrow -5^+} \frac{x^2 - x - 6}{x + 5} = \lim_{x \rightarrow -5^+} \frac{(x-3)(x+2)}{x+5} = \infty$</p> <p>$\frac{x^2 - x - 6}{x + 5}$ has a VA at $x = -5$</p>	<p>2. $\lim_{x \rightarrow -2^-} \frac{x-5}{x^2+x-2} = \lim_{x \rightarrow -2^-} \frac{x-5}{(x+2)(x-1)} = -\infty$</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding: 5px;">x</td> <td style="padding: 5px;">$\frac{x-5}{(x+2)(x-1)}$</td> </tr> <tr> <td style="padding: 5px;">-2.1</td> <td style="padding: 5px;">$\frac{-}{(-)(-)} = -$</td> </tr> </table> <p>$\frac{x-5}{x^2+x-2}$ has a VA at $x = -2$</p>	x	$\frac{x-5}{(x+2)(x-1)}$	-2.1	$\frac{-}{(-)(-)} = -$	<p>3. $\lim_{x \rightarrow 2^-} \frac{2x^2+x-3}{x^2-3x+2} = \lim_{x \rightarrow 2^-} \frac{(2x-3)(x+1)}{(x-2)(x-1)} = -\infty$</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding: 5px;">x</td> <td style="padding: 5px;">$\frac{2x^2+x-3}{x^2-3x+2}$</td> </tr> <tr> <td style="padding: 5px;">1.9</td> <td style="padding: 5px;">$\frac{+}{-} = -$</td> </tr> </table> <p>$\frac{2x^2+x-3}{x^2-3x+2}$ has a VA at $x = 2$</p>	x	$\frac{2x^2+x-3}{x^2-3x+2}$	1.9	$\frac{+}{-} = -$
x	$\frac{x-5}{(x+2)(x-1)}$									
-2.1	$\frac{-}{(-)(-)} = -$									
x	$\frac{2x^2+x-3}{x^2-3x+2}$									
1.9	$\frac{+}{-} = -$									

Find each of the following limits at infinity. Show your limit analysis. Then, explain why the result of the limit concurs with your graphical understanding of asymptotic behavior of the rational function.

<p>4. $\lim_{x \rightarrow -\infty} \frac{3x+2-5x^2}{2x^2-3x-1}$</p> $\begin{aligned} &= \lim_{x \rightarrow -\infty} \frac{\frac{3x}{x^2} + \frac{2}{x^2} - \frac{5x^2}{x^2}}{\frac{2x^2}{x^2} - \frac{3x}{x^2} - \frac{1}{x^2}} \\ &= \lim_{x \rightarrow -\infty} \frac{\frac{3}{x} + \frac{2}{x^2} - 5}{2 - \frac{3}{x} - \frac{1}{x^2}} \\ &= \frac{0 + 0 - 5}{2 - 0 - 0} \\ &= -5/2 \end{aligned}$ <p>$\therefore y = -5/2$ is a HA of graph</p>	<p>5. $\lim_{x \rightarrow \infty} \frac{3x+5}{2x^2-3x}$</p> $\begin{aligned} &= \lim_{x \rightarrow \infty} \frac{\frac{3x}{x^2} + \frac{5}{x^2}}{\frac{2x^2}{x^2} - \frac{3x}{x^2}} \\ &= \frac{0 + 0}{2 - 0} \\ &= 0 \end{aligned}$ <p>$\therefore y = 0$ is HA of graph</p>	<p>6. $\lim_{x \rightarrow -\infty} \frac{-2x^2+5}{3x+2}$</p> $\begin{aligned} &= \lim_{x \rightarrow -\infty} \frac{\frac{-2x^2}{x^2} + \frac{5}{x^2}}{\frac{3x}{x^2} + \frac{2}{x^2}} \\ &= \frac{-2(-\infty) + 0}{3 + 0} \\ &= \frac{\infty}{3} \\ &= \infty, \text{ dire} \end{aligned}$ <p>\therefore There is no HA</p>
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Find each of the following limits at infinity. Explain how you arrived at your answer.

7. $\lim_{x \rightarrow -\infty} (-3x^3 - 2x + 4) = -\infty$

The degree is odd and lead coeff is negative so the graph rises to the left

8. $\lim_{x \rightarrow \infty} (2 - x)(x + 2)^3 = -\infty$

The degree is even with negative lead coefficient so graph falls to the right

9. $\lim_{x \rightarrow \infty} \left[\left(\frac{3}{2} \right)^{-x+2} - 2 \right] = -2$



10. $\lim_{x \rightarrow -\infty} \left[-\left(\frac{2}{3} \right)^{-x+1} + 3 \right] = \lim_{x \rightarrow \infty} \left[-\left(\frac{3}{2} \right)^{x-1} + 3 \right]$

$\leftarrow y = 3 \rightarrow$

Find each of the following limits at infinity. What do the results show about the existence of a horizontal asymptote? Justify your reasoning.

11. $\lim_{x \rightarrow -\infty} \frac{2x+1}{\sqrt{x^2-x}}$ $\sqrt{x^2} = -x \quad (x < 0)$

$$\begin{aligned} &= \lim_{x \rightarrow -\infty} \frac{\frac{2x}{x} + \frac{1}{x}}{\sqrt{\frac{x^2}{x^2} - \frac{x}{x^2}}} \\ &= \lim_{x \rightarrow -\infty} \frac{-2 - \frac{1}{x}}{\sqrt{1 - \frac{1}{x}}} \\ &= \frac{-2 - 0}{\sqrt{1 - 0}} \\ &= \frac{-2}{\sqrt{1}} \\ &= -2 \end{aligned}$$

$\therefore y = -2$ is a horizontal asymptote

12. $\lim_{x \rightarrow \infty} \frac{-2x^2+x}{\sqrt{2x^2-3}}$ $\sqrt{x^2} = x \quad (x > 0)$

$$\begin{aligned} &= \lim_{x \rightarrow \infty} \frac{\frac{-2x^2}{x^2} + \frac{x}{x^2}}{\sqrt{\frac{2x^2}{x^2} - \frac{3}{x^2}}} \\ &= \lim_{x \rightarrow \infty} \frac{-2x + 1}{\sqrt{2 - \frac{3}{x^2}}} \\ &= \frac{-2(\infty) + 1}{\sqrt{2 - 0}} \\ &= \frac{-\infty}{\sqrt{2}} \\ &= -\infty \end{aligned}$$

\therefore there is no H.A.