## Unit 2.10 Finding Derivatives of $\tan(x)$, $\cot(x)$, $\sec(x)$ and $\csc(x)$

**Find the derivative of each function.**

| 1. $y = 4x - \tan x$ | 2. $h(x) = 3x \cot x$ | 3. $r = \dfrac{\theta}{\tan \theta}$ |
|---|---|---|

1. $y = 4x - \tan x$

$y' = 4 - \sec^2(x)$

2. $h(x) = 3x \cot x$

$h'(x) = 3 \cdot \cot(x) + 3x(-\csc^2(x))$

$= 3\cot(x) - 3x \csc^2(x)$

3. $r = \dfrac{\theta}{\tan \theta}$

$r' = \dfrac{1 \cdot \tan\theta - \theta \cdot \sec^2\theta}{(\tan\theta)^2}$

$r' = \dfrac{\tan\theta - \theta \cdot \sec^2\theta}{\tan^2\theta}$ ✓

$r' = \dfrac{\tan\theta}{\tan^2\theta} - \theta \cdot \dfrac{\sec^2\theta}{\tan^2\theta}$ ✓

$r' = \dfrac{1}{\tan\theta} - \theta \dfrac{\frac{1}{\cos^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta}}$ ✓

$r' = \cot\theta - \theta \dfrac{1}{\sin^2\theta}$ ✓

$r' = \cot\theta - \theta\csc^2\theta$ ✓

*Any of these could be MC answers*

4. $g(x) = 4\sec x - \ln x$

$g'(x) = 4\sec x \tan x - \dfrac{1}{x}$

5. $y = -5\csc x$

$\dfrac{dy}{dx} = -5(-\csc x \cdot \cot x)$

$\dfrac{dy}{dx} = 5\csc x \cot x$

## Find the derivative at the given $x$-value.  Show your work!

**6.**  $f(x) = 2 \cot x$ at $x = \frac{3\pi}{4}$.

$f'(x) = 2(-\csc^2 x)$

$f'\left(\frac{3\pi}{4}\right) = -2\left[\csc\left(\frac{3\pi}{4}\right)\right]^2$

$= -2\left[\sqrt{2}\right]^2$

$= -2 \cdot 2$

$f'\left(\frac{3\pi}{4}\right) = -4$
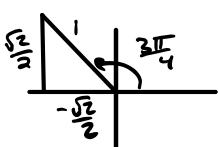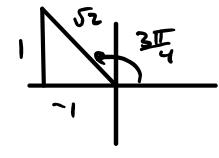
**7.**  $f(x) = \csc x$ at $x = \frac{\pi}{3}$.

$f'(x) = -\csc x \cot x$

$f'\left(\frac{\pi}{3}\right) = -\csc\left(\frac{\pi}{3}\right) \cdot \cot\left(\frac{\pi}{3}\right)$

$= -\frac{2}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}}$

$f'\left(\frac{\pi}{3}\right) = \frac{-2}{3}$
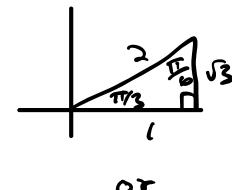
## Estimate the derivative at the given $x$-value by using a calculator.

**8.**  $f(x) = \cot^2\left(\frac{x}{5}\right)$ at $x = 0.2$.

$f'(0.2) \approx -6{,}250.000$

| Plot 1 | Plot 2 | Plot 3 |

$\blacksquare \; \backslash Y_1 \; \blacksquare \; \left(\dfrac{1}{\tan\left(\frac{X}{5}\right)}\right)^2$

$\dfrac{d}{dX}(Y_1)_{X=.2}$

$-6{,}249.99965$

**9.**  $f(x) = -x \sec(7x)$ at $x = 1$.

$f'(1) \approx -9.418$

| Plot 1 | Plot 2 | Plot 3 |

$\blacksquare \; \backslash Y_1 \; \blacksquare \; -X \dfrac{1}{\cos(7X)}$

$\dfrac{d}{dX}(Y_1)_{X=1}$

$-9.41785$
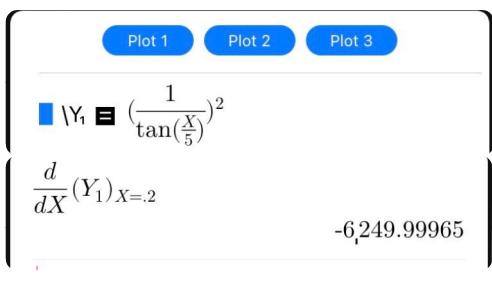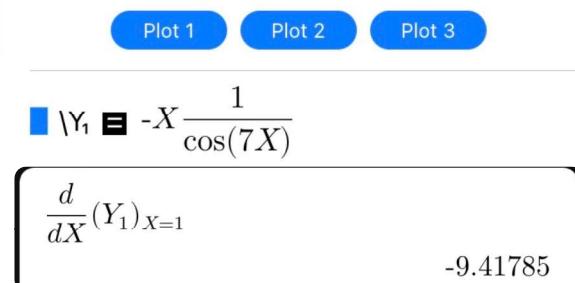
### Answers to 2.10 CA #1

| 1.  $4 - \sec^2 x$ | 2.  $3\cot x - 3x\csc^2 x$ | 3.  $\dfrac{\tan\theta - \theta\sec^2\theta}{\tan^2\theta} = \cot\theta - \csc^2\theta$ | 4.  $4\sec x \tan x - \dfrac{1}{x}$ |
|---|---|---|---|
| 5.  $5\csc x \cot x$ | 6.  $-4$ | 7.  $-\dfrac{2}{3}$ | 8.  $-6250.311$ \| 9.  $-9.4185$ |