Name ~~Answer Key~~ Date______ Class______

Day #12 Homework

Find $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ for each of the functions below. Then, find the equation of the tangent line to the graph of $f(x)$ at the given value of x.

1. $f(x) = x^3 + 2x$

$\lim_{h\to 0} \frac{(x+h)^3+2(x+h)-(x^3+2x)}{h}$

$\lim_{h\to 0} \frac{(x+h)(x^2+2xh+h^2)+\cancel{2x}+2h-x^3\cancel{-2x}}{h}$

$\lim_{h\to 0} \frac{\cancel{x^3}+2x^2h+xh^2+x^2h+2xh^2+h^3+2h\cancel{-x^3}}{h}$

$\lim_{h\to 0} \frac{3x^2h+3xh^2+h^3+2h}{h}$

$\lim_{h\to 0} \frac{\cancel{h}(3x^2+3xh+h^2+2)}{\cancel{h}}$

$3x^2+3x(0)+(0)^2+2$

$3x^2+2$

$\boxed{f'(x) = 3x^2+2}$

3. $f(x) = \sqrt{3-x}$

$\lim_{h\to 0} \frac{\sqrt{3-(x+h)}-\sqrt{3-x}}{h}$

$\lim_{h\to 0} \frac{\sqrt{3-x-h}-\sqrt{3-x}}{h} \cdot \frac{\sqrt{3-x-h}+\sqrt{3-x}}{\sqrt{3-x-h}+\sqrt{3-x}}$

$\lim_{h\to 0} \frac{3-x-h-(3-x)}{h(\sqrt{3-x-h}+\sqrt{3-x})}$

$\lim_{h\to 0} \frac{\cancel{3}-\cancel{x}-h\cancel{-3}+\cancel{x}}{\cancel{h}(\sqrt{3-x-h}+\sqrt{3-x})}$

$\lim_{h\to 0} \frac{-1}{\sqrt{3-x-h}+\sqrt{3-x}}$

$\frac{-1}{\sqrt{3-x-0}+\sqrt{3-x}}$

$\boxed{f'(x) = \frac{-1}{2\sqrt{3-x}}}$

2. Find the equation of the line tangent to the graph of $f(x) = x^3 + 2x$ at $x = -1$.

P.O.T: $f(-1) = (-1)^3+2(-1)$
$= -1-2 = -3$

S.O.T: $f'(-1) = 3(-1)^2+2$
$= 3+2 = 5$

$\boxed{y+3 = 5(x+1)}$

4. Find the equation of the line tangent to the graph of $f(x) = \sqrt{3-x}$ at $x = -6$.

P.O.T: $f(-6) = \sqrt{3-(-6)} = \sqrt{9} = 3$

S.O.T: $f'(-6) = \frac{-1}{2\sqrt{3-(-6)}}$
$= \frac{-1}{2(3)} = -\frac{1}{6}$

$\boxed{y-3 = -\frac{1}{6}(x+6)}$

For problems 5 – 9, use the function $f(x)=\dfrac{x}{x+2}$.

5. Find $f'(x)$ by finding $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$.

$$\lim_{h\to 0}\frac{\dfrac{x+h}{x+h+2}-\dfrac{x}{x+2}}{h}$$

$$\lim_{h\to 0}\frac{(x+h)(x+2)-x(x+h+2)}{(x+h+2)(x+2)}\cdot\frac{1}{h}$$

$$\lim_{h\to 0}\frac{\cancel{x^2}+\cancel{2x}+xh+2h-\cancel{x^2}-xh-\cancel{2x}}{h(x+h+2)(x+2)}$$

$$\lim_{h\to 0}\frac{\cancel{h}(x+2-x)}{\cancel{h}(x+h+2)(x+2)}$$

$$\lim_{h\to 0}\frac{2}{(x+h+2)(x+2)}$$

$$\frac{2}{(x+0+2)(x+2)}$$

$$\boxed{f'(x)=\frac{2}{(x+2)^2}}$$

6. Find the slope of the tangent line drawn to the graph of $f(x)$ at $x=-2$.

$$f'(-2)=\frac{2}{(-2+2)^2}=\frac{2}{0}$$

$$=\boxed{\text{undefined}}$$

7. Find the slope of the tangent line drawn to the graph of $f(x)$ at $x=-1$.

$$f'(-1)=\frac{2}{(-1+2)^2}=\frac{2}{1}=\boxed{2}$$

8. Find the equation of the tangent line drawn to the graph of $f(x)$ at $x=-1$.

$$f(-1)=\frac{-1}{-1+2}=\frac{-1}{1}=-1$$

$$f'(-1)=2$$

$$\boxed{y+1=2(x+1)}$$

9. Find $\lim\limits_{x\to a}\dfrac{f(x)-f(a)}{x-a}$, where $a=-1$.

$$\lim_{x\to -1}\frac{\dfrac{x}{x+2}-(-1)}{x+1}$$

$$\lim_{x\to -1}\frac{\dfrac{x}{x+2}+\dfrac{x+2}{x+2}}{x+1}$$

$$\lim_{x\to -1}\frac{2x+2}{x+2}\cdot\frac{1}{x+1}$$

$$\lim_{x\to -1}\frac{2\cancel{(x+1)}}{x+2}\cdot\frac{1}{\cancel{x+1}}=\frac{2}{-1+2}$$

$$=\frac{2}{1}$$

$$=\boxed{2}$$