

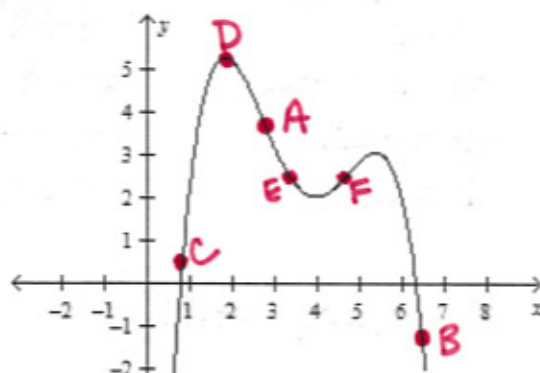
Day #13 Homework

1. The line defined by the equation $2y + 3 = -\frac{2}{3}(x - 3)$ is tangent to the graph of $g(x)$ at $x = -3$. What is the value of $\lim_{x \rightarrow -3} \frac{g(x) - g(-3)}{x + 3}$? Show your work and explain your reasoning.

The value of $\lim_{x \rightarrow -3} \frac{g(x) - g(-3)}{x + 3}$ is the slope of the tangent line drawn to the graph of $g(x)$ at $x = -3$. Therefore, the slope of $2y + 3 = -\frac{2}{3}(x - 3)$ is the value of the given limit.

$$2y + 3 = -\frac{2}{3}x + 2 \quad 2y = -\frac{2}{3}x - 1 \quad y = -\frac{1}{3}x - \frac{1}{2} \quad \boxed{-\frac{1}{3}}$$

Use the graph of $f(x)$ pictured to the right to perform the actions in exercises 2 – 6. Give written explanations for your choices.

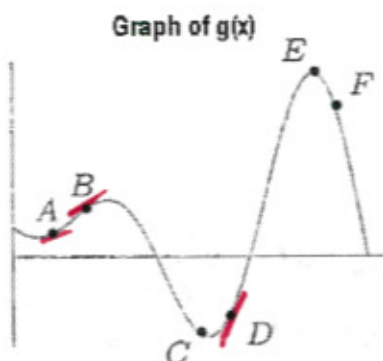


Sample placement of points A through F.

- Label a point, A, on the graph of $y = f(x)$ where the derivative is negative.
Point A should be placed on the graph where $f(x)$ is decreasing if $f'(A) < 0$.
- Label a point, B, on the graph of $y = f(x)$ where the value of the function is negative.
Point B should be placed on the graph where $f(x)$ is below the x-axis.
- Label a point, C, on the graph of $y = f(x)$ where the derivative is greatest in value.
Point C should be placed on the graph where $f(x)$ is increasing and where the tangent line drawn to $f(x)$ is the steepest.
- Label a point, D, on the graph of $y = f(x)$ where the derivative is zero.
Point D should be labeled at a relative maximum or minimum in order for the derivative to be 0.
- Label two different points, E and F, on the graph of $y = f(x)$ where the values of the derivative are opposites.
Points E and F should be placed on the graph, one where the function is increasing and the other where the function is decreasing, and so that their tangent lines are sloping at approximately the same (opposite) angle.

7. Match the points on the graph of $g(x)$ with the value of $g'(x)$ in the table.

Value of $g'(x)$	Point on $g(x)$
-3	F
-1	C
0	E
$\frac{1}{2}$	A
1	B
2	D



8. The function to the right is such that $h(4) = 25$ and $h'(4) = 1.5$. Find the coordinates of A , B , and C .

$$A = (4, 25)$$

$$B = (4.2, B)$$

$$C = (3.9, C)$$

$$\frac{25 - B}{4 - 4.2} = 1.5$$

$$25 - B = -0.3$$

$$-B = -25.3$$

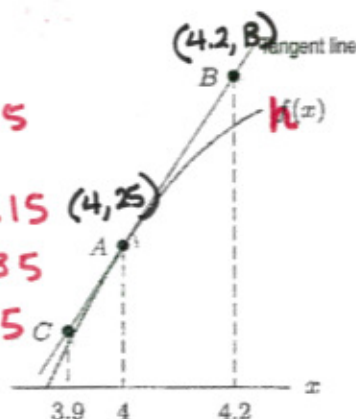
$$B = 25.3$$

$$\frac{25 - C}{4 - 3.9} = 1.5$$

$$25 - C = 0.15$$

$$-C = -24.85$$

$$C = 24.85$$



$$B = (4.2, 25.3)$$

$$C = (3.9, 24.85)$$

For exercises 9 – 11, use the function $f(x) = \frac{1}{x+1}$.

9. Find $f'(x)$.

$$\lim_{h \rightarrow 0} \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h}$$

$$\lim_{h \rightarrow 0} \frac{x+1 - x-h-1}{(x+1)(x+h+1)} \cdot \frac{1}{h}$$

$$\lim_{h \rightarrow 0} \frac{-1}{(x+1)(x+h+1)} = \frac{-1}{(x+1)(x+1)}$$

$$f'(x) = -\frac{1}{(x+1)^2}$$

10. Find the equation of the tangent line drawn to the graph of $f(x)$ at $x = 0$.

$$F(0) = \frac{1}{0+1} = 1$$

$$F'(0) = -\frac{1}{(0+1)^2} = -1$$

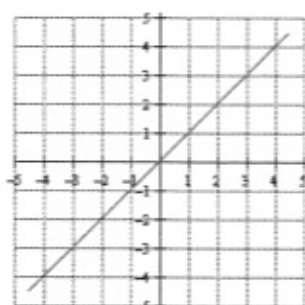
$$y - 1 = -1(x - 0)$$

11. Find the equation of the normal line drawn to the graph of $f(x)$ at $x = 0$.

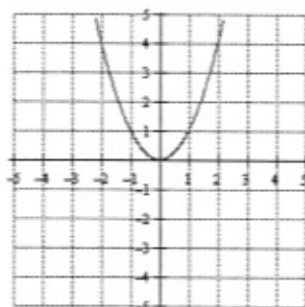
$$\text{Slope of Normal line} = 1$$

$$y - 1 = 1(x - 0)$$

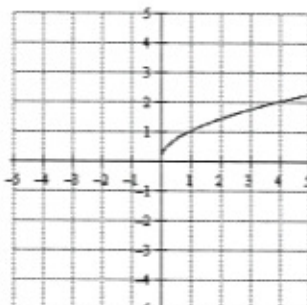
12. Given below are graphs of four functions— $f(x)$, $g(x)$, $h(x)$, and $p(x)$. Below those graphs are graphs of their derivatives. Label the graphs below as $f'(x)$, $g'(x)$, $h'(x)$, and $p'(x)$.



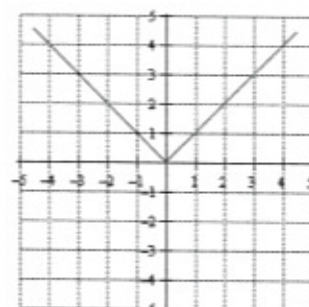
$f(x)$



$g(x)$

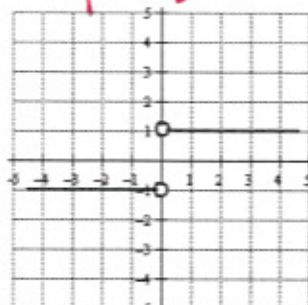


$h(x)$

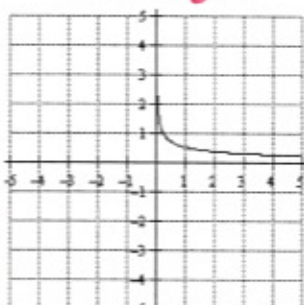


$p(x)$

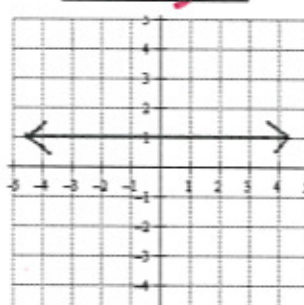
$p'(x)$



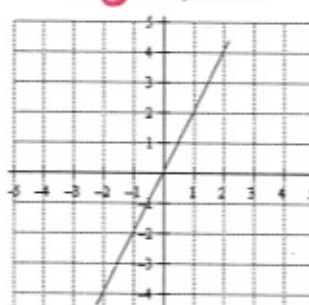
$h'(x)$



$f'(x)$



$g'(x)$



The table below represents values on the graph of a cubic polynomial function, $h(x)$. Use the table to complete exercises 22 – 24.

x	-3	-2	-1	0	1	2	4
$h(x)$	-24	0	8	6	0	-4	18

13. Two of the zeros of $h(x)$ are listed in the table. Between which two values of x does the Intermediate Value Theorem guarantee that a third value of x exists such that $h(x) = 0$? Explain your reasoning.

$h(x)$ must have a zero between $x=2$ and $x=4$
 b/c $h(2) = -4$ means the graph is below the x -axis
 and $h(4) = 18$ means the graph is above the x -axis
 so at an intermediate value between $x=2$ and $x=4$, the graph must cross the x -axis.

14. Estimate the value of $h'(1.5)$. Based on this value, describe the behavior of $h(x)$ at $x = 1.5$. Justify your reasoning.

$$h'(1.5) \approx \frac{h(1) - h(2)}{1 - 2} \approx \frac{0 - 4}{-1} = 4$$

$h'(1.5) \approx -4$ which means at $x = 1.5$, $h(x)$ should be decreasing since the derivative is < 0 .

15. Estimate the value of $h'(-1.75)$. Based on this value, describe the behavior of $h(x)$ at $x = -1.75$. Justify your reasoning.

$$h'(-1.75) \approx \frac{h(-2) - h(-1)}{-2 - (-1)} \approx \frac{0 - 8}{-1} = 8$$

$h'(-1.75) \approx 8$ which means at $x = -1.75$, $h(x)$ should be increasing since the derivative is > 0 .