Unit 2.2 Defining Derivative of a Function and Using Derivative Notation

Find the derivative using limits. If the equation is given as $y =$, use Leibniz Notation: $\frac{dy}{dx}$. If the equation is given as $f(x) =$, use Lagrange Notation: $f'(x)$. WRITE SMALL!!

1. $y = x^2 + 2x - 9$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{[(x+h)^2 + 2(x+h) - 9] - \{x^2 + 2x - 9\}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{2x} + 2h \cancel{- 9} - \cancel{x^2} - \cancel{2x} \cancel{+ 9}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2x + h + 2)}{\cancel{h}}$$

$$= 2x + (0) + 2$$

$$\frac{dy}{dx} = 2x + 2$$

2. $f(x) = \frac{1}{5-x}$

$$f'(x) = \lim_{h\to 0} \frac{\frac{1}{5-(x+h)} \cdot (5-x)\cancel{(5-x-h)} - \frac{1}{5-x} \cdot \cancel{(5-x)}(5-x-h)}{h \cdot (5-x)(5-x-h)}$$

$$= \lim_{h\to 0} \frac{(5-x) - (5-x-h)}{h \cdot (5-x)(5-x-h)}$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(5-x)(5-x-h)}$$

$$= \frac{1}{(5-x)(5-x-(0))}$$

$$f'(x) = \frac{1}{(5-x)^2}$$

3. $y = \sqrt{4x - 1}$

$$\frac{dy}{dx} = \frac{\sqrt{4(x+h) - 1} - \sqrt{4x-1}}{h} \cdot \frac{(\sqrt{4(x+h) - 1} + \sqrt{4x-1})}{(\sqrt{4(x+h) - 1} + \sqrt{4x-1})}$$

$$\frac{dy}{dx} = \frac{(4(x+h) - 1) - (4x - 1)}{h(\sqrt{4(x+h) - 1} + \sqrt{4x-1})}$$

$$\frac{dy}{dx} = \frac{\cancel{4x} + 4h \cancel{- 1} \cancel{- 4x} \cancel{+ 1}}{h(\sqrt{4(x+h) - 1} + \sqrt{4x-1})}$$

$$\frac{dy}{dx} = \frac{4\cancel{h}}{\cancel{h}(\sqrt{4(x+h) - 1} + \sqrt{4x-1})}$$

$$\frac{dy}{dx} = \frac{4}{\sqrt{4(x+0) - 1} + \sqrt{4x-1}}$$

$$\frac{dy}{dx} = \frac{4}{2\sqrt{4x-1}} = \frac{2}{\sqrt{4x-1}}$$

For each problem, create an equation of the tangent line of f at the given point.

4. $f(1) = -5$ and $f'(1) = 3$

PoT SoT

$$y - (-5) = 3(x - 1)$$

5. $f(x) = x \sin x$
$f'(x) = \sin x + x \cos x$; $x = \pi$

PoT	SoT	Tangent
$f(\pi) = \pi \sin(\pi)$ $= \pi(0)$ $f(\pi) = 0$	$f'(\pi) = \sin\pi + \pi\cos\pi$ $= 0 + \pi(-1)$ $= -\pi$	$y - 0 = -\pi(x - \pi)$

For each problem, create an equation of the tangent line of f at the given point.

6. $f(x) = \sqrt{5x+1}$

$f'(x) = \frac{5}{\sqrt{5x+1}}$; $x = 7$

PoT	SoT	Tangent
$f(7) = \sqrt{5(7)+1}$ $f(7) = 6$	$f'(7) = \frac{5}{\sqrt{5(7)+1}} = \frac{5}{6}$	$y - 6 = \frac{5}{6}(x-7)$

For each problem, use the information given to identify the meaning of the two equations in the context of the problem. Write in full sentences!

7. $p(h)$ is the number of people standing in line at a popular amusement park and h is the number of hours since the park opened.

hours

$p(2) = 2005$ and $p'(3) = 250$ people/hour

people hours

- At 2 hours, there are 2005 people in line
- At 3 hours, the # of people in line is increasing at a rate of 250 people per hour

8. $W(t)$ is the volume of water (in liters) in a backpack during a hike and t is the number of minutes passed since the hike began.

$W(30) = 2.1$ and $d'(90) = -0.07$ $\frac{\text{liters}}{\text{min}}$

min liters min

- At 30 minutes, the backpacks have 2.1 liters of H_2O
- At 90 min, the water is <u>decreasing</u> at a rate of 0.07 liters/min

Answers to 2.2 CA #1

1. $2x+2$	2. $\frac{1}{(5-x)^2}$	3. $\frac{2}{\sqrt{4x-1}}$	4. $y+5 = 3(x-1)$	5. $y = -\pi(x-\pi)$	6. $y-6 = \frac{5}{6}(x-7)$

7. After 2 hours, there are 2005 people in line. On the 3rd hour, the number of people in line is increasing by 250 people per hour.	8. After 30 minutes, 2.1 liters of water is in the backpack. At the 90-minute mark, the water is decreasing at a rate of 0.07 liters per minute.