Name Answer Key Date________ Class________

**Day #14 Homework**

For exercises 1 – 12, find the derivative of each function. Leave your answers with no negative or rational exponents and as single rational functions, when applicable.

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| 1. $f(x) = 5 - 2x^2 - 3x^3$<br>$\boxed{f'(x) = -4x - 9x^2}$ | 2. $h(x) = \frac{2x^3 + 3x^2 - 2x}{x} = 2x^2 + 3x - 2$<br>$\boxed{h'(x) = 4x + 3}$ |
| 3. $h(x) = \frac{3}{x^7} = 3x^{-7}$<br>$h'(x) = -21x^{-8}$<br>$\boxed{h'(x) = -\frac{21}{x^8}}$ | 4. $g(x) = \frac{2x^5}{x^8} = 2x^{-3}$<br>$g'(x) = -6x^{-4}$<br>$\boxed{g'(x) = -\frac{6}{x^4}}$ |
| 5. $f(\theta) = -3\theta^2 - \cos\theta$<br>$f'(\theta) = -6\theta - (-\sin\theta)$<br>$\boxed{f'(\theta) = -6\theta + \sin\theta}$ | 6. $h(x) = \sqrt[3]{x^2} = x^{2/3}$<br>$h'(x) = \frac{2}{3}x^{-1/3}$<br>$h'(x) = \frac{2}{3} \cdot \frac{1}{x^{1/3}}$<br>$\boxed{h'(x) = \frac{2}{3\sqrt[3]{x}}}$ |
| 7. $g(\theta) = \sqrt{\theta} + 2\sin\theta = \theta^{\frac{1}{2}} + 2\sin\theta$<br>$g'(\theta) = \frac{1}{2}\theta^{-1/2} + 2\cos\theta$<br>$g'(\theta) = \frac{1}{2\sqrt{\theta}} + 2\cos\theta$<br>$\boxed{g'(\theta) = \frac{1 + 4\sqrt{\theta}\cos\theta}{2\sqrt{\theta}}}$ | 8. $p(x) = -2x^{\frac{3}{2}} + \sqrt{x} = -2x^{3/2} + x^{1/2}$<br>$p'(x) = -3x^{1/2} + \frac{1}{2}x^{-1/2}$<br>$p'(x) = -3\sqrt{x} + \frac{1}{2\sqrt{x}}$<br>$\boxed{p'(x) = \frac{-6x + 1}{2\sqrt{x}}}$ |

| | |
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| 9. $g(x) = (x+3)(2x-1)^2$<br>$g(x) = (x+3)(4x^2-4x+1)$<br>$g(x) = 4x^3 - 4x^2 + x$<br>$+12x^2 - 12x + 3$<br>$g(x) = 4x^3 + 8x^2 - 11x + 3$<br>$\boxed{g'(x) = 12x^2 + 16x - 11}$ | 10. $h(x) = \frac{x^2+2x-2}{x^3} = x^{-1} + 2x^{-2} - 2x^{-3}$<br>$h'(x) = -1x^{-2} - 4x^{-3} + 6x^{-4}$<br>$h'(x) = -\frac{1}{x^2} - \frac{4}{x^3} + \frac{6}{x^4}$<br>$h'(x) = \frac{-x^2 - 4x + 6}{x^4}$<br>$\boxed{h'(x) = -\frac{x^2+4x-6}{x^4}}$ |
| 11. $f(x) = \frac{3x}{\sqrt[3]{x}} = 3x \cdot x^{-1/3}$<br>$f(x) = 3x^{2/3}$<br>$f'(x) = 2x^{-1/3}$<br>$\boxed{f'(x) = \frac{2}{\sqrt[3]{x}}}$ | 12. $h(x) = 6\sqrt{x} - 3\cos x = 6x^{1/2} - 3\cos x$<br>$h'(x) = 3x^{-1/2} - 3(-\sin x)$<br>$h'(x) = \frac{3}{\sqrt{x}} + 3\sin x$<br>$\boxed{h'(x) = \frac{3 + 3\sqrt{x}\sin x}{\sqrt{x}}}$ |

13. For what value(s) of $x$ will the slope of the tangent line to the graph of $h(x) = 4\sqrt{x}$ be $-2$? Find the equation of the line tangent to $h(x)$ at this/these $x$ – values. Show your work.

$h(x) = 4x^{1/2}$

$h'(x) = 2x^{-\frac{1}{2}}$

$h'(x) = \frac{2}{\sqrt{x}}$

$\frac{2}{\sqrt{x}} = -2$

$-2\sqrt{x} = 2$

$\sqrt{x} = -1$

No solution

Since $h(x)$ is always increasing, the slope of the tangent line will never be negative.

14. Find the equation of the line tangent to the graph of $g(x)=\frac{2}{\sqrt[4]{x^3}}$ when $x=1$.

$g(1)=\frac{2}{\sqrt[4]{1^3}}=2$

POT : $(1,2)$

SOT : $-\frac{3}{2}$

$\boxed{y-2=-\frac{3}{2}(x-1)}$

$g(x)=2x^{-3/4}$

$g'(x)=-\frac{3}{2}x^{-7/4}$

$g'(x)=-\frac{3}{2\sqrt[4]{x^7}}$

$g'(1)=-\frac{3}{2\sqrt[4]{1}}=-\frac{3}{2}$

15. The line defined by the equation $\frac{1}{2}x+3=-2(y-3)$ is the line tangent to the graph of a function $f(x)$ when $x=a$. What is the value of $f'(a)$? Show your work and explain your reasoning.

$f'(a)$ is the slope of the tangent line so the slope of $\frac{1}{2}x+3=-2(y-3)$ is $f'(a)$. $f'(a)=-\frac{1}{4}$

$\frac{1}{2}x+3=-2y+6$

$-\frac{1}{2}[-2y=\frac{1}{2}x-3$

$y=-\frac{1}{4}x+\frac{3}{2}$

16. The line defined by the equation $y-3=-\frac{2}{3}(x+3)$ is the line tangent to the graph of a function $f(x)$ at the point $(-3, 3)$. What is the equation of the normal line when $x=-3$. Explain your reasoning.

Slope of the normal line and tangent line are opposite reciprocals and they pass through the same point on the graph of $f(x)$.

$f'(-3)$ = slope of tangent $=-\frac{2}{3}$ so the slope of the normal line is $\frac{3}{2}$.

$\boxed{y-3=\frac{3}{2}(x+3)}$

17. Determine the value(s) of $x$ at which the function $f(x)=x^4-8x^2+2$ has a horizontal tangent.

$f'(x)=4x^3-16x=0$

$4x(x^2-4)=0$

$4x(x+2)(x-2)=0$

$\boxed{x=0 \quad x=-2 \quad x=2}$

18. Determine the value(s) of $\theta$ at which the function $f(\theta) = \sqrt{3}\theta + 2\cos\theta$ has a horizontal tangent on the interval $[0, 2\pi)$.

$f(\theta) = \sqrt{3}\theta + 2\cos\theta$

$f'(\theta) = \sqrt{3} - 2\sin\theta = 0$

$-2\sin\theta = -\sqrt{3}$

$\sin\theta = \frac{\sqrt{3}}{2}$

$\boxed{\theta = \frac{\pi}{3} \text{ and } \frac{2\pi}{3}}$

19. For what value(s) of $k$ is the line $y = 4x - 9$ tangent to the graph of $f(x) = x^2 - kx$?

$f'(x) = 2x - K = 4$

$2x - 4 = K$

$x = 3$

$K = 2(3) - 4$

$K = 6 - 4$

$\boxed{K = 2}$

$x^2 - Kx = 4x - 9$

$x^2 - x(2x - 4) = 4x - 9$

$x^2 - 2x^2 + 4x = 4x - 9$

$-x^2 = -9$

$x^2 = 9$

$x = \pm 3$

$x = -3$

$K = 2(-3) - 4$

$K = -6 - 4$

$\boxed{K = -10}$