Name Answer Key Date_____ Class_____

Day #15 Homework

For exercises 1 – 3, determine on what intervals the given function is increasing or decreasing. Also, identify the coordinates of any relative extrema of the function. Show your work and justify your reasoning.

1. $f(x) = 2x^3 + 3x^2 - 12x$

$f'(x) = 6x^2 + 6x - 12 = 0$

$6(x^2 + x - 2) = 0$

$6(x+2)(x-1) = 0$

$f'(x)$: $x = -3$ | $x = 0$ | $x = 3$

$(+)(-)(-)$ | -2 | $(+)(-)(+)$ | 1 | $(+)(+)(+)$

Pos. | Neg | Pos.

$2(-2)^3 + 3(-2)^2 - 12(-2)$
$-16 + 12 + 24$
20

- $f(x)$ is increasing when $f'(x) > 0$ which occurs on the interval $(-\infty, -2) \cup (1, \infty)$. $f(x)$ is decreasing when $f'(x) < 0$ which occurs on the interval $(-2, 1)$.
- $f(x)$ has a relative max @ $(-2, 20)$ b/c $f'(x)$ changes from + to − at $x = -2$. $f(x)$ has a relative minimum at $(1, -7)$ b/c $f'(x)$ changes from − to + @ $x = 2$.

2. $g(x) = x^3 - 6x^2 + 15$

$g'(x) = 3x^2 - 12x = 0$

$3x(x-4) = 0$

$x = 0 \quad x = 4$

$g'(x)$: $x = -2$ | $x = 2$ | $x = 5$

$(-)(-)$ | 0 | $(+)(-)$ | 4 | $(+)(+)$

Pos | Neg | Pos

- $f(x)$ is increasing on $(-\infty, 0) \cup (4, \infty)$ b/c $f'(x) > 0$ and $f(x)$ is decreasing on $(0, 4)$ b/c $f'(x) < 0$.
- $f(x)$ has a relative max at $(0, 15)$ b/c $f'(x)$ changes from pos to neg at $x = 0$.
- $f(x)$ has a relative min at $(4, -17)$ b/c $f'(x)$ changes from neg to pos at $x = 4$.

3. $h(x) = (x+2)^2(x-1)$

$h(x) = (x-1)(x^2+4x+4)$

$h(x) = x^3+4x^2+4x-x^2-4x-4$

$h(x) = x^3+3x^2-4$

$h'(x) = 3x^2+6x = 0$

$3x(x+2) = 0$

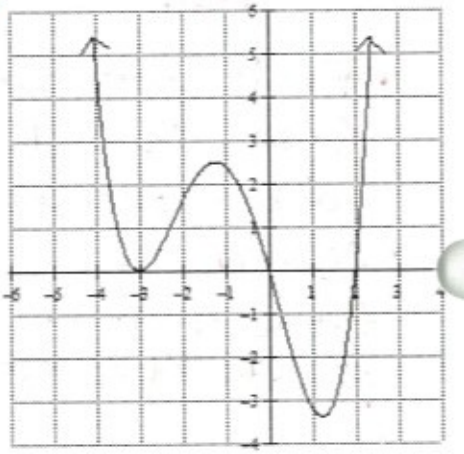
$x=0 \quad x=-2$

$f'(x)$: $x=-4$ | $x=-1$ | $x=2$

$(-)(-)$ Pos | -2 | $(-)(+)$ Neg | 0 | $(+)(+)$ Pos

- $h(x)$ is increasing on $(-\infty,-2)\cup(0,\infty)$ b/c $h'(x)>0$
- $h(x)$ is decreasing on $(-2,0)$ b/c $h'(x)<0$.
- $h(x)$ has a relative max @ $(-2,0)$ b/c $h'(x)$ changes from pos to neg at $x=-2$.
- $h(x)$ has a relative min @ $(0,-4)$ b/c $h'(x)$ changes from neg to pos at $x=0$

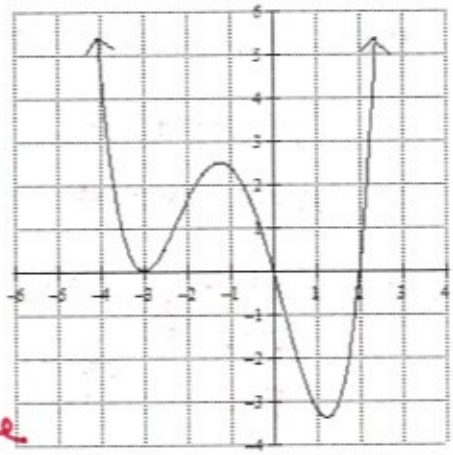
4. Pictured to the right is the graph of $f'(x)$. On what interval(s) is the graph of $f(x)$ increasing or decreasing? Justify your reasoning.



$f(x)$ is increasing when $f'(x)>0$ which occurs on the interval $(-\infty,-3)\cup(-3,0)\cup(2,\infty)$.

$f(x)$ is decreasing when $f'(x)<0$ which occurs on the interval $(0,2)$.

5. Pictured to the right is the graph of $f'(x)$. At what value(s) of x does the graph of $f(x)$ have a relative maximum/minimum? Justify your reasoning.



$f(x)$ has a relative max when $f'(x)$ changes from positive to negative which occurs at $x=0$.

$f(x)$ has a relative minimum when $f'(x)$ changes from negative to positive which occurs at $x=2$.