Name **Date** **Class**

Homework 2.5

1. If $g'(x) = (x-3)^2(x+1)$, determine on what intervals the graph of $g(x)$ is increasing or decreasing and identify the value(s) of x at which $g(x)$ has a relative maximum or minimum. Justify your reasoning and show your work.

CV: $x = -1, 3$

$$\boxed{\begin{aligned} g'(x) &= (x-3)^2(x+1) \\ 0 &= (x-3)^2(x+1) \\ 0 &= (x-3)^2 \quad \Big\} \quad 0 = x+1 \\ 0 &= x-3 \qquad\quad -1 = x \\ 3 &= x \end{aligned}}$$

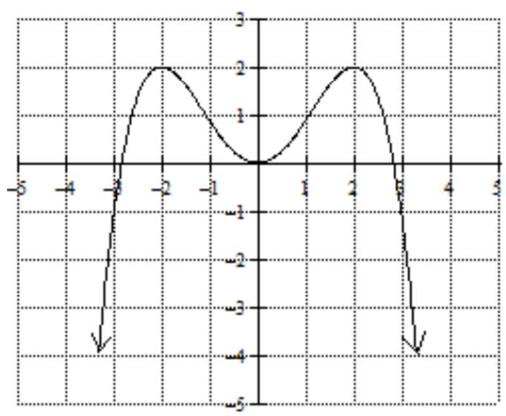
$(+)(-) = -$ | $(+)(+) = +$ | $(+)(+) = +$

$f'(x)$: $x=-2$ ... -1 ... $x=0$ ... 3 ... $x=4$

neg | pos | pos

- $g(x)$ is increasing on $(-1,3) \cup (3,\infty)$ b/c $g' > 0$ on these intervals
- $g(x)$ is decreasing on $(-\infty,-1)$ b/c $g' < 0$ on this interval
- $g(x)$ has a relative minimum at $x=-1$ b/c g' changes from negative to positive at $x=-1$
- $g(x)$ does not have a relative maximum b/c g' never changes from positive to negative.

For exercises 2 – 4, use the graph of the function, $h(x)$, pictured to the right. Use the graph to identify the following. **Provide written justification.**



2. On what interval(s) is $h'(x) < 0$?

$h' < 0$ on $(-2,0) \cup (2,\infty)$

b/c h is decreasing on those intervals

3. On what interval(s) is $h'(x) > 0$?

$h' > 0$ on $(-\infty,-2) \cup (0,2)$

b/c h is increasing on those intervals

4. At what value(s) of x does $h'(x)$ change from positive to negative? From negative to positive?

- $h'(x)$ changes from positive to negative at $x=-2, 2$

  b/c $h(x)$ changes from increasing to decreasing at $x=-2,2$ or

  b/c $h(x)$ has a relative max at $x=-2,2$

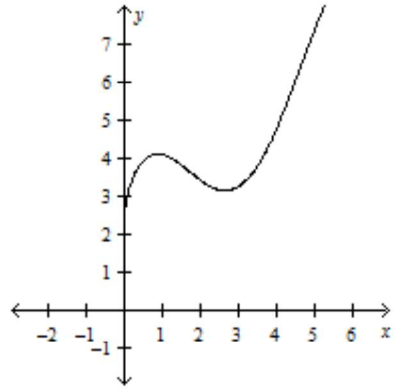
- $h'(x)$ changes from negative to positive at $x=0$

  b/c $h(x)$ changes from decreasing to increasing at $x=0$ or

  b/c $h(x)$ has a relative min at $x=0$

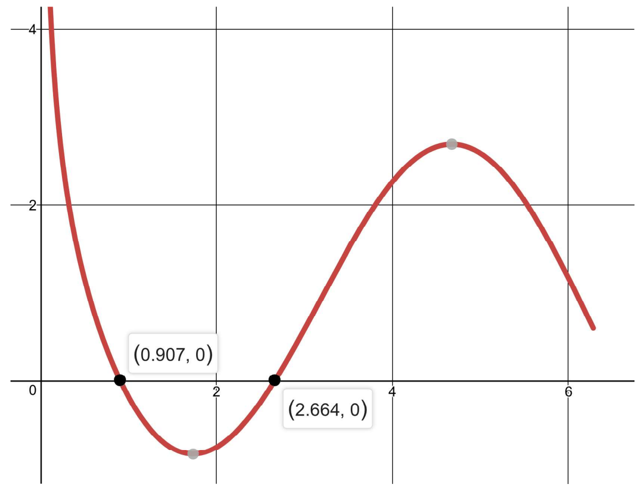
Consider the function, $g(x)=3\sqrt{x}+2\cos x$, which is pictured to the right on the interval $0<x\le 2\pi$. $g(x)=3x^{\frac{1}{2}}+2\cos x$

Graph of $g(x)$



5. Algebraically find $g'(x)$. Express your answer as a single rational function with positive exponents.

$$g' = \frac{3}{2}x^{-\frac{1}{2}} - 2\sin x$$
$$= \frac{3}{2\sqrt{x}} - 2\sin x \cdot \frac{2\sqrt{x}}{2\sqrt{x}}$$
$$g' = \frac{3-4\sqrt{x}\sin x}{2\sqrt{x}}$$



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6. Sketch the graph of $g'(x)$ on the axes to the right. Then, state the value(s) of x where the graph of $g(x)$ has a relative maximum and/or minimum. Justify your answers.

- $g(x)$ has a rel max at $x=0.907$ b/c $g'(x)$ changes from positive to negative at $x=0.907$
- $g(x)$ has a rel min at $x=2.664$ b/c $g'(x)$ changes from negative to positive at $x=2.664$

7. Based on the graph of $g(x)$, will the slope of the tangent line be positive or negative at $x = 4.5$. Give a reason for your answer.

- $g(x)$ will have a tangent line with a positive slope at $x=4.5$ b/c $g(x)$ is increasing at $x=4.5$

8. Use the equation of $g'(x)$ to verify your answer to exercise 7.

$g'(4.5)=\frac{3-4\sqrt{4.5}\sin(4.5)}{2\sqrt{4.5}}>0$, which confirms excercise 7.

9. Find the equation of the tangent line to the graph of $g(x)$ when $x = 4.5$.

$y-5.942 = 2.662(x-4.5)$

calculator

10. Use the equation of the tangent line at $x = 4.5$ to approximate $g(4.1)$. Then, use the equation of g to find $g(4.1)$.Is the tangent line approximation an under or over approximation? Explain why this is true based on the graph.

$$\boxed{\begin{aligned} y-5.942 &= 2.662[(4.1)-4.5] \\ y &= 2.662[-0.4]+5.942 \\ y &= 4.8772 \end{aligned}}$$

$g(4.1)=4.8772$, which is an underapproximation because the graph of $g(x)$ is concave up at $x=4.1$

11. For what function does $\lim_{h\to 0}\dfrac{2\sin(x+h)-2\sin x}{h}$ give the derivative? Find the limit.

$\lim_{h\to 0}\dfrac{2\sin(x+h)-2\sin x}{h}$ is the derivative of the function $f(x)=2\sin x$. The limit is the same as $f'(x)$.

Therefore $\lim_{h\to 0}\dfrac{2\sin(x+h)-2\sin x}{h} = 2\cos x$

12. Find $\lim_{h\to 0}\dfrac{(x+h)^5-x^5}{h}$. $= 5x^4$

$f(x)= x^5$

$f'(x) = 5x^4$

13. Find $\lim_{h\to 0}\dfrac{\sqrt{x}-\sqrt{x+h}}{h}$.

$\sqrt{x} \to x^{\frac{1}{2}}$

$= \lim_{h\to 0}\dfrac{-\sqrt{x+h}-(-\sqrt{x})}{h} = -\frac{1}{2}x^{-\frac{1}{2}} = -\dfrac{1}{2\sqrt{x}}$

$f(x) = -\sqrt{x}$

$f'(x) = \dfrac{-1}{2\sqrt{x}}$

14. If $f(x)=\dfrac{3x}{\sqrt{x}}$, what is the slope of the normal line to the graph of $f(x)$ when $x = 4$?

SoT

$f(x) = 3x\cdot x^{-\frac{1}{2}} = 3x^{\frac{1}{2}}$

$f'(x) = \frac{3}{2}x^{-\frac{1}{2}} = \dfrac{3}{2\sqrt{x}}$

$f'(4) = \dfrac{3}{2\sqrt{4}} = \dfrac{3}{2\cdot 2} = \dfrac{3}{4}$

SoN:

$\boxed{m = -4/3}$

$-4/3$ is the slope of the normal line to the graph of $g(x)$ at $x=4$.

15. If $2x - 3 = 5(y + 1)$ is the equation of the normal line to the graph of $f(x)$ when $x = a$, find the value of $f'(a)$. Show your work and explain your reasoning.

$2(x-\frac{3}{2}) = 5(y+1)$

$\frac{2}{5}(x-\frac{3}{2}) = y+1$

$y-(-1) = \frac{2}{5}(x-\frac{3}{2})$

PoT: $(\frac{3}{2},-1)$ overkill

SoN: $\frac{2}{5}$

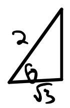
SoT $= -5/2$

$f'(a)$ represents the slope of the tangent line when $x=a$. By writing the equation of the normal line in point-slope form, we can see the slope of the normal line is $\frac{2}{5}$. Therefore the tangent line's slope is $-5/2$.

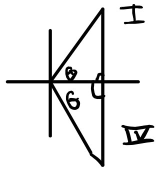
16. On the interval $[0, 2\pi)$, find the coordinates of the relative minimum(s) of $f(\theta) = \sqrt{3}\theta - 2\sin\theta$.

CV:

①
$$\boxed{\begin{aligned} f'(\theta) &= \sqrt{3} - 2\cos\theta \\ 0 &= \sqrt{3} - 2\cos\theta \\ 2\cos\theta &= \sqrt{3} \\ \cos\theta &= \frac{\sqrt{3}}{2} \\ \theta &= \frac{\pi}{6}, \frac{11\pi}{6} \end{aligned}}$$



$\Rightarrow \theta = 30° = \frac{\pi}{6}$



③ CP: $\left(\frac{\pi}{6}, \frac{\pi\sqrt{3}}{6} - 1\right)$

$$\boxed{\begin{aligned} f\left(\tfrac{\pi}{6}\right) &= \sqrt{3}\cdot\frac{\pi}{6} - 2\sin\left(\frac{\pi}{6}\right) \\ &= \frac{\pi\sqrt{3}}{6} - 2\left(\frac{1}{2}\right) \\ f\left(\tfrac{\pi}{6}\right) &= \frac{\pi\sqrt{3}}{6} - 1 \end{aligned}}$$

②

	$\sqrt{3} - 2\cos(0)$ $= \sqrt{3} - 2\cdot 1$ $= -$ $\theta = 0$		$\sqrt{3} - 2\cos(\pi)$ $= \sqrt{3} - 2(-1)$ $= +$ $\theta = \pi$		$\sqrt{3} - 2\cos(2\pi)$ $= \sqrt{3} - 2\cdot 1$ $= -$ $\theta = 2\pi$
$f'(\theta)$	neg	$\frac{\pi}{6}$	pos	$\frac{11\pi}{6}$	neg

④ • $f(\theta)$ has a relative min at $\left(\frac{\pi}{6}, \frac{\pi\sqrt{3}}{6} - 1\right)$ b/c $f'(\theta)$ changes from neg. to pos. at $x = \frac{\pi}{6}$

The derivative of a function $f(x)$ is $f'(x) = (3-x)^2(x+5)$. Use this derivative for exercises 17 and 18.

17. At what value(s) of x does the graph of $f(x)$ have a relative maximum? Justify your answer.

CV: $x = -5, 3$

$$\boxed{\begin{aligned} 0 &= (3-x)^2(x+5) \\ 0 &= (3-x)^2, \quad 0 = x+5 \\ 0 &= 3-x \qquad -5 = x \\ x &= 3 \end{aligned}}$$

	$(+)(-)$		$(+)(+)$		$(+)(+)$
$f'(x)$	$x = -6$	-5	$x = 0$	3	$x = 6$
	neg		pos		pos

• $f(x)$ does not have a relative maximum b/c $f'(x)$ never changes from positive to negative.

18. Use the equation of the tangent line to approximate the value of $f(2.1)$ if $f(2) = -3$. If it is known that $f(x)$ is concave down at $x = 2$, is this approximation an over or under approximation of $f(2.1)$? Give a reason for your answer.

PoT = $(2, -3)$

SoT = 7

$$\boxed{\begin{aligned} f'(2) &= (3-2)^2(2+5) \\ &= (-1)^2(7) \\ &= 1\cdot 7 \\ f'(2) &= 7 \end{aligned}}$$

$y + 3 = 7(x-2)$

$y = 7x - 14 - 3$

$y = 7x - 17$

$$\boxed{\begin{aligned} y(2.1) &= 7(2.1) - 17 \\ &= 14.7 - 17 \\ y(2.1) &= -2.3 \end{aligned}}$$

• $f(2.1) \approx -2.3$ which is an over approximation for $f(2.1)$ b/c the graph is concave down at $x = 2.1$