Unit 2.6 Derivative Rules – Constant, Sum, Difference & Constant Multiple Rules

Find the derivative of each function.

1. $y = 3x^{100} - 2x^8 - 7x$

$\frac{dy}{dx} = 300x^{99} - 16x^7 - 7$

2. $V(r) = \frac{4}{3}\pi r^3$

$V'(r) = 4\pi r^2$

3. $f(x) = \frac{1}{x^3} + \frac{12}{x}$ $= x^{-3} + 12x^{-1}$

$\frac{df}{dx} = -3x^{-4} - 12x^{-2}$

$\frac{df}{dx} = \frac{-3}{x^4} - \frac{12}{x^2}$

4. $y = \sqrt[3]{x^2} + 8\sqrt[4]{x^7}$ $= x^{2/3} + 8x^{7/4}$

$y' = \frac{2}{3}x^{-\frac{1}{3}} + 14x^{3/4}$

$y' = \frac{2}{3\sqrt[3]{x}} + 14\sqrt[4]{x^3}$

5. $y = \frac{1}{x^3} - \frac{1}{2}x^4 + ex^2$ $= x^{-3} - \frac{1}{2}x^4 + ex^2$

$\frac{dy}{dx} = -3x^{-4} - 2x^3 + 2ex$

$\frac{dy}{dx} = \frac{-3}{x^4} - 2x^3 + 2ex$

Find the x-value(s) where the function has a horizontal tangent. $y' = 0$ (slope $= 0$)

6. $f(x) = \frac{x^3}{3} - 3x^2 + 9x - 10$ $= \frac{1}{3}x^3 - 3x^2 + 9x - 10$

$f'(x) = x^2 - 6x + 9$

tangent is horizontal when $f'(x) = 0$

$0 = x^2 - 6x + 9$

$0 = (x-3)^2$

f has a horizontal tangent when $x = 3$.

7. $f(x) = \frac{x^4}{4} + x^3 + x^2 + 1$ $= \frac{1}{4}x^4 + x^3 + x^2 + 1$

$f'(x) = x^3 + 3x^2 + 2x$

tangent is horizontal when $f'(x) = 0$

$0 = x(x^2 + 3x + 2)$

$0 = x(x+2)(x+1)$

f has a horizontal tangent when $x = -2, -1, 0$

Find the equations of the tangent AND normal lines of each function at the given value of x. $\perp f'$

8. $y = x^2 + 6x + 9$ at $x = -2$

POT/PON $(-2, 1)$

$y(-2) = (-2)^2 + 6(-2) + 9$

$= 4 - 12 + 9$

$y(-2) = 1$

SOT

$y'(x) = 2x + 6$

$y'(-2) = 2(-2) + 6$

$y'(-2) = 2$

Tangent: $y - 1 = 2(x+2)$

Normal: $y - 1 = -\frac{1}{2}(x+2)$

9. $f(x) = -2\sqrt{x} + 4x$ at $x = 9$

$f(x) = -2x^{\frac{1}{2}} + 4x$

POT/PON

$f(9) = -2\sqrt{9} + 4(9)$

$= -6 + 36$

$f(9) = 30$

SOT

$f'(x) = -1 \cdot x^{-\frac{1}{2}} + 4$

$f'(9) = \frac{-1}{\sqrt{9}} + 4$

$f'(9) = -\frac{1}{3} + 4$

$= -\frac{1}{3} + \frac{12}{3}$

$f'(9) = \frac{11}{3}$

Tangent: $y - 30 = \frac{11}{3}(x - 9)$

Normal: $y - 30 = \frac{-3}{11}(x - 9)$

Are the functions differentiable at the given value of x?

10. At $x = 2$.

$12x^{-2} - 3x + 2$

$$f(x) = \begin{cases} \frac{12}{x^2} - 3x + 2, & x < 2 \\ x^3 - 3x^2 + 3, & x \geq 2 \end{cases}$$

Continuous at $x = 2$?

I. $f(2) = 2^3 - 3(2)^2 + 3$

$= 8 - 12 + 3$

$f(2) = -1$ $\therefore f(2)$ is defined

II. $\lim_{x \to 2^-} f(x) = -1 = \lim_{x \to 2^+} f(x)$

$\therefore \lim_{x \to 2} f(x)$ exists

III. $f(2) = \lim_{x \to 2} f(x)$

$\therefore f$ is continuous at $x = 2$

Diff @ $x = 2$

$$f' = \begin{cases} -24x^{-1} - 3, & x < 2 \\ 3x^2 - 6x, & x \geq 2 \end{cases}$$

$x \to 2^-$

$f'(2) = \frac{-24}{2} - 3 = -15$

$x \to 2^+$

$f'(2) = 3(2)^2 - 6(2) = 0$

$f(x)$ is not differentiable at $x = 2$

11. At $x = 1$.

$$f(x) = \begin{cases} x - \frac{3}{x}, & x \leq 1 \\ 2x - \frac{4}{\sqrt{x}}, & x > 1 \end{cases} \Rightarrow \frac{df}{dx} = \begin{cases} 1 + \frac{3}{x^2} \\ 2 + \frac{2}{\sqrt{x^3}} \end{cases}$$

Cont @ $x = 1$?

I. $f(1) = 1 - 3 = -2$

$\therefore f(1)$ is defined

II. $\lim_{x \to 1^-} f(x) = -2 = \lim_{x \to 1^+} f(x)$

$\therefore \lim_{x \to 1} f(x)$ exists

III. $\lim_{x \to 1} f(x) = f(1)$

$\therefore f$ is continuous @ $x = 1$

Diff @ $x = 1$?

$x \to 1^-$

$\left.\frac{df}{dx}\right|_{x=1} = 1 + \frac{3}{1^2} = 4$

$x \to 1^+$

$\left.\frac{df}{dx}\right|_{x=1} = 2 + \frac{2}{\sqrt{1^3}} = 4$

f is differentiable at $x = 1$

What values of a and b would make the function differentiable at the given value of x?

12. At $x = -1$

$$f(x) = \begin{cases} x^3 - ax + 4, & x \leq -1 \\ ax^2 + bx - 2, & x > -1 \end{cases}$$

Continuous

$\lim_{x \to -1^-} (x^3 - ax + 4) = \lim_{x \to -1^+} (ax^2 + bx - 2)$

$(-1)^3 - a(-1) + 4 = a(-1)^2 + b(-1) - 2$

$-1 + a + 4 = a - b - 2$

$+a + 3 = a - b - 2$

$b = -5$

Diff

$$f'(x) = \begin{cases} 3x^2 - a, & x \leq -1 \\ 2ax + b & x > -1 \end{cases}$$

$3(-1)^2 - a = 2a(-1) + b$

$3 - a = -2a + b$

$3 = -a + b$

$3 = -a + (-5)$

$8 = -a$

$-8 = a$

$(a, b) = (-8, -5)$

13. At $x = 1$.

$$f(x) = \begin{cases} ax^2 + bx, & x < 1 \\ x - b, & x \geq 1 \end{cases} \Rightarrow f' = \begin{cases} 2ax + b \\ 1 \end{cases}$$

Cont @ $x = 1$

$a(1)^2 + b(1) = 1 - b$

$a + b = 1 - b$

$a = 1 - 2b$

Diff @ $x = 1$

$2a(1) + b = 1$

$2a + b = 1$

$2(1 - 2b) + b = 1$

$2 - 4b + b = 1$

$-3b = -1$

$b = \frac{1}{3}$

$a = 1 - 2(\frac{1}{3})$

$a = \frac{3}{3} - \frac{2}{3}$

$a = \frac{1}{3}$

Answers to 2.6 CA #1

1. $\frac{dy}{dx} = 300x^{99} - 16x^7 - 7$	2. $V'(r) = 4\pi r^2$	3. $f'(x) = -\frac{3}{x^4} - \frac{12}{x^2}$	4. $\frac{dy}{dx} = \frac{2}{3\sqrt[3]{x}} + 14\sqrt[4]{x^3}$
5. $\frac{dy}{dx} = -\frac{3}{x^4} - 2x^3 + 2ex$	6. $x = 3$	7. $x = 0, -2, -1$	8. T: $y - 1 = 2(x + 2)$ N: $y - 1 = -\frac{1}{2}(x + 2)$

9. T: $y - 30 = \frac{11}{3}(x - 9)$ N: $y - 30 = -\frac{3}{11}(x - 9)$	10. No. Continuous, but not differentiable.	11. Yes.	12. $a = -8$, $b = -5$	13. $a = \frac{1}{3}$, $b = \frac{1}{3}$