Unit 2.7 Finding Derivatives of $\sin(x)$, $\cos(x)$, $\ln(x)$ and e^x

Find the derivative of each function.

1.) $f(\theta)=\frac{\pi}{2}\sin\theta-\cos\theta$

$f'(\theta)=\frac{\pi}{2}\cos\theta+\sin\theta$

2.) $y=x^2-\frac{1}{2}\cos x$

$y'=2x+\frac{1}{2}\sin x$

3.) $f(x)=\frac{1}{2}e^x-3\sin x$

$\frac{df}{dx}=\frac{1}{2}e^x-3\cos x$

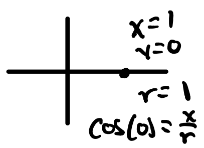
4.) $f(x)=\frac{1}{x^2}-2e^x = x^{-2}-2e^x$

$\frac{df}{dx}=-2x^{-3}-2e^x$

$\frac{df}{dx}=\frac{-2}{x^3}-2e^x$

Find the slope of the graph of the function at the given point. Use proper notation.

5.) $f(\theta)=4\sin\theta-\theta,\ (0,0)$



$\frac{df}{d\theta}=4\cos\theta-1$

$\left.\frac{df}{d\theta}\right|_{(0,0)}=4\cos(0)-1$

$=4(1)-1$

$\left.\frac{df}{d\theta}\right|_{(0,0)}=3$

6.) $f(x)=\frac{3}{4}e^x,\ \left(0,\frac{3}{4}\right)$

$f'(x)=\frac{3}{4}e^x$

$f'(0)=\frac{3}{4}e^0$

$f'(0)=\frac{3}{4}$

Write the equation of the tangent line to the graph of the function at the given point

7.) $f(x) = x + e^x$, $(0,1)$ PoT

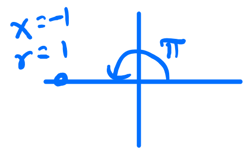
SoT

$f'(x) = 1 + e^x$

$f'(0) = 1 + e^0$

$f'(0) = 2$

$y - 1 = 2(x - 0)$ or $y = 1 + 2x$



PoT

8.) $g(t) = \sin t + \frac{1}{2}e^t$, $\left(\pi, \frac{1}{2}e^\pi\right)$

$\cos(\pi) = \frac{x}{r}$

$g'(t) = \cos(t) + \frac{1}{2}e^t$

$g'(\pi) = \cos(\pi) + \frac{1}{2}e^\pi$

$g'(\pi) = -1 + \frac{1}{2}e^\pi$

SoT

$y - \frac{1}{2}e^\pi = \underbrace{\left(-1 + \frac{1}{2}e^\pi\right)}_{\text{Slope}}(x - \pi)$

Determine the point(s) (if any) at which the graph of the function has a horizontal tangent line. (Hint: a horizontal tangent line has a slope of 0)

9.) $f(x) = -4x + e^x$

$f'(x) = -4 + e^x$

$f' = 0$

$0 = -4 + e^x$

$4 = e^x$

$\ln 4 = \ln e^x$

$\ln 4 = x$

f has a horizontal tangent when $x = \ln 4$

10.) $g(x) = x + \sin x$, $0 \le x < 2\pi$

$g'(x) = 1 + \cos x$

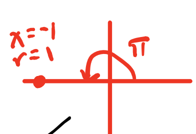
$g'(x) = 0$

$0 = 1 + \cos x$

$-1 = \cos x$

$\cos^{-1}(-1) = x$

$x = \pi$



f has a horizontal tangent when $x = \pi$

For problem 11, use proper notation throughout.

11.) Consider the function $f(t) = \ln t$.

a) Calculate the instantaneous rate of change of the function at $t = \frac{1}{2}$.

f'

$f'(t) = \frac{1}{t}$

$f'(\frac{1}{2}) = \frac{1}{\frac{1}{2}} \cdot \frac{2}{2} = 2$

$f'(\frac{1}{2}) = 2$

b) Find the equation of the tangent line at the point where $t = 3$. Leave your answer in terms of the natural logarithm.

PoT	SoT	Tangent line
$f(3) = \ln(3)$	$f'(3) = \frac{1}{3}$	$y - \ln(3) = \frac{1}{3}(x - 3)$

Recall the various versions of the definition of derivative from Topics 2.1-2.3:

$f'(x)=\lim\limits_{\Delta x\to 0}\dfrac{f(x+\Delta x)-f(x)}{\Delta x}$, $f'(c)=\lim\limits_{h\to 0}\dfrac{f(c+h)-f(c)}{h}$ and $f'(c)=\lim\limits_{x\to c}\dfrac{f(x)-f(c)}{x-c}$

12.) Find the following using the Limit Definition of Derivative. You should be able to do this with very little computation.

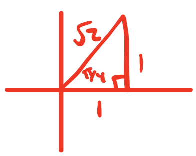
a.) $\lim\limits_{\Delta x\to 0}\dfrac{\sin(x+\Delta x)-\sin(x)}{\Delta x}$ $=\dfrac{d}{dx}(\sin x)=\cos x$

b.) $\lim\limits_{h\to 0}\dfrac{(x+h)^2-x^2}{h}$ $=\dfrac{d}{dx}(x^2)=2x$

c.) $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\sin x-\sin\frac{\pi}{4}}{x-\frac{\pi}{4}}$ $=\dfrac{d}{dx}(\sin x)\Big|_{x=\frac{\pi}{4}}=\cos(x)\Big|_{x=\frac{\pi}{4}}$

$=\cos\left(\frac{\pi}{4}\right)$

$=\frac{\sqrt{2}}{2}$



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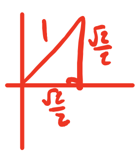
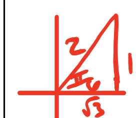


d.) $\lim\limits_{h\to 0}\dfrac{\sin\left(\frac{\pi}{6}+h\right)-\frac{1}{2}}{h}$ $=\dfrac{d}{dx}(\sin x)\Big|_{x=\frac{\pi}{6}}$

$=\cos(x)\Big|_{x=\frac{\pi}{6}}$

$=\cos\left(\frac{\pi}{6}\right)$

$=\frac{\sqrt{3}}{2}$



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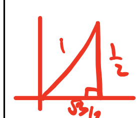

e.) $\lim\limits_{\Delta x\to 0}\dfrac{(2+\Delta x)^3-8}{\Delta x}$ $=\dfrac{d}{dx}(x^3)\Big|_{x=2}=3x^2\Big|_{x=2}$

$=3(2)^2$

$=12$

f.) $\lim\limits_{x\to\pi}\dfrac{\cos x+1}{x-\pi}$ $=\dfrac{d}{dx}(\cos x)\Big|_{x=\pi}=-\sin(x)\Big|_{x=\pi}$

$=-\sin(\pi)$

$=-(0)$

$=0$

g.) $\lim\limits_{x\to 2}\dfrac{\ln x-\ln 2}{x-2}$ $=\dfrac{d}{dx}(\ln x)\Big|_{x=2}=\dfrac{1}{x}\Big|_{x=2}=\dfrac{1}{2}$

h.) $\lim\limits_{\Delta x\to 0}\dfrac{(3+\Delta x)^2+(3+\Delta x)-12}{\Delta x}$ $=\dfrac{d}{dx}(x^2+x)\Big|_{x=3}$

$=(2x+1)\Big|_{x=3}$

$=2(3)+1$

$=7$