Unit 2.8 The Product Rule

Find the derivative of each function.

1. $h(x) = 5x^2 \ln x$

$h'(x) = 10x \cdot \ln(x) + 5x^2 \cdot \frac{1}{x}$

$h'(x) = 10x \cdot \ln(x) + 5x$

$h'(x) = 5x\left[2\ln(x) + 1\right]$

GCF

2. $h(x) = 3e^x(2 - 7x)$

GCF GCF

$\frac{dh}{dx} = 3e^x(2-7x) + 3e^x(-7)$

GCF

$\frac{dh}{dx} = 3e^x\left[2-7x-7\right]$

$\frac{dh}{dx} = 3e^x\left[-7x-5\right]$

$\frac{dh}{dx} = -3e^x\left[7x+5\right]$

3. $f(x) = 6\sin x \cos x + x$

$\frac{df}{dx} = 6\cos x \cos x + 6\sin x(-\sin x) + 1$

$\frac{df}{dx} = 6\cos^2 x - 6\sin^2 x + 1$

4. $g(x) = \frac{5}{x}\cos x = 5x^{-1}\cos x$

$g'(x) = -5x^{-2} \cdot \cos x + 5x^{-1}(-\sin x)$

$g'(x) = \frac{-5}{x^2}\cos x - \frac{5}{x}\sin x$

$g'(x) = -\frac{5}{x}\left(\frac{1}{x}\cos x + \sin x\right)$

GCF

5. $f(x) = \frac{e^x}{2}\ln x = \frac{1}{2}e^x \cdot \ln x$

$\frac{df}{dx} = \frac{1}{2}e^x \cdot \ln x + \frac{1}{2}e^x \cdot \frac{1}{x}$

$\frac{df}{dx} = \frac{1}{2}e^x\left[\ln x + \frac{1}{x}\right]$

GCF

Use the table to find the value of the derivatives of each function.

6.

x	$j(x)$	$j'(x)$	$k(x)$	$k'(x)$
-1	3	-4	5	-6

a. $h(x) = 2j(x)k(x)$
Find $h'(-1)$.

$h'(x) = 2\cdot j'(x)\cdot k(x) + 2j(x)\cdot k'(x)$

$h'(-1) = 2\cdot j'(-1)\cdot k(-1) + 2j(-1)\cdot k'(-1)$

$= 2(-4)\cdot 5 + 2(3)\cdot(-6)$

$= -40 - 36$

$h'(-1) = -76$

b. $f(x) = \left(\frac{j(x)}{2} - 4\right)(1 - k(x)) = \left(\frac{1}{2}j - 4\right)(1 - k)$
Find $f'(-1)$.

$f'(x) = \left(\frac{1}{2}j'(x)\right)\cdot(1 - k(x)) + \left(\frac{1}{2}j(x) - 4\right)(-k'(x))$

$f'(-1) = \frac{1}{2}\cdot j'(-1)\cdot(1 - k(-1)) + \left(\frac{1}{2}j(-1) - 4\right)(-k'(-1))$

$= \frac{1}{2}(-4)(1-5) + \left(\frac{1}{2}(3) - 4\right)(-(-6))$

$= 8 + \left[\frac{3}{2} - 4\right](6)$

$= 8 + 9 - 24$

$f'(-1) = -7$

Use the table to find the value of the derivatives of each function.

7.

t	$c(t)$	$c'(t)$	$l(t)$	$l'(t)$
2	4	-2	-1	2

a. $f(t) = -3c(t)l(t)$
Find $f'(2)$.

$f'(t) = -3c'(t)\,l(t) - 3c(t)\cdot l'(t)$

$f'(2) = -3c'(2)\,l(2) - 3c(2)\,l'(2)$

$= -3\cdot(-2)(-1) - 3(4)(2)$

$= -6 - 24$

$f'(2) = -30$

b. $h(t) = (5 - c(t))(1 + 2l(t))$
Find $h'(2)$.

$h'(t) = -c'(t)\cdot[1 + 2l(t)] + [5 - c(t)][2l'(t)]$

$h'(2) = -c'(2)[1 + 2l(2)] + [5 - c(2)][2l'(2)]$

$= -(-2)[1 + 2(-1)] + [5 - 4][2\cdot 2]$

$= 2[-1] + [1][4]$

$h'(2) = 2$

Answers to 2.8 CA #1

1. $10x\ln x + 5x$	2. $-3e^x(5 + 7x)$	3. $6\cos^2 x - 6\sin^2 x + 1$	4. $-\frac{5\cos x}{x^2} - \frac{5\sin x}{x}$	
5. $\frac{e^x \ln x}{2} + \frac{e^x}{2x}$	6a. -76	6b. -7	7a. -30	7b. 2