Unit 2.9 The Quotient Rule

Find the derivative of each function.

1. $h(x) = \frac{x^2+4x-1}{x-3}$

$h'(x) = \frac{(2x+4)(x-3) - (x^2+4x-1)(1)}{(x-3)^2}$

$h'(x) = \frac{2x^2-6x+4x-12-x^2-4x+1}{(x-3)^2}$

$h'(x) = \frac{x^2-6x-11}{(x-3)^2}$

2. $g(x) = \frac{5e^x}{2\sin x}$

$\frac{dg}{dx} = \frac{5e^x \cdot 2\sin x - 5e^x \cdot 2\cos x}{(2\sin x)^2}$

$\frac{dg}{dx} = \frac{10e^x[\sin x - \cos x]}{4\sin^2 x}$

3. $f(x) = \frac{3\cos x}{2-7x}$

$f'(x) = \frac{-3\sin x(2-7x) - 3\cos x(-7)}{(2-7x)^2}$

$f'(x) = \frac{21\cos x - 3\sin x(2-7x)}{(2-7x)^2}$

4. $g(x) = \frac{4x^3-5x^2+5x}{x^2} = \frac{4x^3}{x^2} - \frac{5x^2}{x^2} + \frac{5x}{x^2}$

$= 4x - 5 + 5x^{-1}$

$g'(x) = 4 - 5x^{-2}$

$g'(x) = 4 - \frac{5}{x^2}$

5. $h(x) = \frac{5x^2}{\ln x}$

$\frac{dh}{dx} = \frac{10x \cdot \ln x - 5x^2 \cdot \frac{1}{x}}{(\ln x)^2}$

$= \frac{10x \cdot \ln x - 5x}{(\ln x)^2}$

$\frac{dh}{dx} = \frac{5x[2\ln x - 1]}{(\ln x)^2}$

6. $f(x) = \frac{x+2}{x^2+2}$

$\frac{df}{dx} = \frac{1(x^2+2) - (x+2)2x}{(x^2+2)^2}$

$= \frac{x^2+2-2x^2-4x}{(x^2+2)^2}$

$\frac{df}{dx} = \frac{-x^2-4x+2}{(x^2+2)^2}$

$\frac{df}{dx} = -\frac{x^2+4x-2}{(x^2+2)^2}$

7. $h(x) = \frac{8x^3+4x^2-9x}{2x} = \frac{8x^3}{2x} + \frac{4x^2}{2x} - \frac{9x}{2x}$

$= 4x^2 + 2x + \frac{9}{2}$

$h'(x) = 8x + 2$

Use the table to find the value of the derivatives of each function.

8.

x	$j(x)$	$j'(x)$	$k(x)$	$k'(x)$
-1	6	-4	5	-6

a. $h(x) = \frac{2j(x)}{k(x)}$

Find $h'(-1)$.

$h'(x) = \frac{2j'(x)\cdot k(x) - 2j(x)\cdot k'(x)}{[k(x)]^2}$

$h'(-1) = \frac{2j'(-1)k(-1) - 2j(-1)k'(-1)}{[k(-1)]^2}$

$= \frac{2(-4)5 - 2(6)(-6)}{(5)^2}$

$= \frac{-40 + 72}{25}$

$h'(-1) = \frac{32}{25}$

b. $f(x) = \frac{1-k(x)}{\frac{j(x)}{2}-4}$

Find $f'(-1)$.

$f'(x) = \frac{-k'(x)\left(\frac{1}{2}j(x)-4\right) - (1-k(x))\cdot\frac{1}{2}j'(x)}{\left[\frac{1}{2}j(x)-4\right]^2}$

$f'(-1) = \frac{-k'(-1)\left(\frac{1}{2}\cdot j(-1)-4\right) - (1-k(-1))\cdot\frac{1}{2}j'(-1)}{\left[\frac{1}{2}j(-1)-4\right]^2}$

$= \frac{-(-6)\left[\frac{1}{2}(6)-4\right] - [1-5]\cdot\frac{1}{2}(-4)}{\left[\frac{1}{2}(6)-4\right]^2}$

$= \frac{6[-1] - [-4][-2]}{[-1]^2}$

$= \frac{-6-8}{1}$

$f'(-1) = -14$

Use the table to find the value of the derivatives of each function.

9.

t	$c(t)$	$c'(t)$	$l(t)$	$l'(t)$
2	3	-2	-1	2

a. $f(t) = -\frac{l(t)}{3c(t)} = \frac{-l(t)}{3c(t)}$

Find $f'(2)$.

$f'(t) = \frac{-l'(t)\cdot 3c(t) - [-l(t)](3c'(t))}{[3c(t)]^2}$

$f'(2) = \frac{-l'(2)\cdot 3c(2) + l(2)\,3\cdot c'(2)}{[3\cdot c(2)]^2}$

$= \frac{-2\cdot 3\cdot 3 + (-1)\cdot 3(-2)}{[3\cdot 3]^2}$

$= \frac{-18+6}{81}$

$= \frac{-12}{81}$

$f'(2) = \frac{-4}{27}$

b. $h(t) = \frac{5-c(t)}{1+2l(t)}$

Find $h'(2)$.

$h'(t) = \frac{-c'(t)(1+2l(t)) - (5-c(t))\cdot 2l'(t)}{[1+2l(t)]^2}$

$h'(2) = \frac{-c'(2)[1+2l(2)] - [5-c(2)]\cdot 2l'(2)}{[1+2l(2)]^2}$

$= \frac{-(-2)[1+2(-1)] - [5-3]\cdot 2(2)}{[1+2(-1)]^2}$

$= \frac{2(-1) - (2)4}{(-1)^2}$

$= \frac{-2-8}{1}$

$h'(2) = -10$

Answers to 2.9 CA #1

1. $\frac{x^2-6x-11}{(x-3)^2}$	2. $\frac{10e^x(\sin x-\cos x)}{4\sin^2 x}$	3. $\frac{21\cos x-3\sin x(2-7x)}{(2-7x)^2}$	4. $4-\frac{5}{x^2}$
5. $\frac{10x\ln x-5x}{(\ln x)^2}$	6. $-\frac{x^2+4x-2}{(x^2+2)^2}$	7. $8x+2$	8a. $\frac{32}{25}$ 8b. -14
			9a. $-\frac{4}{27}$ 9b. -10