

## Homework 3.5

Find the derivative of each of the following functions.

<p>1. <math>f(x) = 3^{x^2+2x}</math></p> <p style="text-align: center;">Exp Function Rule</p> $\dot{f}(x) = 3^{x^2+2x} \cdot (2x+2) \cdot \ln 3$ $\dot{f}(x) = (2x+2) 3^{x^2+2x} \ln 3$	<p>2. <math>h(x) = 5^x \sin x</math> Product Rule</p> <p style="text-align: center;">Exp. func. Rule</p> $h'(x) = 5^x \cdot (1) \cdot \ln 5 \cdot \sin x + 5^x \cos x$ $h'(x) = 5^x (\ln 5 \sin x + \cos x)$
<p>3. <math>g(x) = 3x(2^{-x})</math></p> <p style="text-align: center;">Product Rule</p> <p style="text-align: center;">Exp. func. Rule</p> $g'(x) = 3(2^{-x}) + 3x \cdot (2^{-x}) \cdot (-1) \cdot \ln 2$ $g'(x) = 3(2^{-x}) [1 - x \ln 2]$	<p>4. <math>p(x) = \frac{2^x}{2x}</math></p> <p style="text-align: center;">Exp. func. Rule</p> $p'(x) = \frac{2^x (1) \ln 2 \cdot 2x - 2^x (2)}{(2x)^2}$ <p style="text-align: right;">} Quotient Rule</p> $= \frac{(2^x) 2(x \ln 2 - 1)}{4x^2}$ $p'(x) = \frac{2^x (x \ln 2 - 1)}{2x^2}$
<p>5. <math>g(x) = \frac{3^x}{x^3}</math></p> <p style="text-align: center;">Exp. func. Rule</p> $g'(x) = \frac{3^x (1) \ln 3 \cdot x^3 - 3^x (3x^2)}{(x^3)^2}$ <p style="text-align: right;">} Quotient Rule</p> $g'(x) = \frac{(3^x)(x^2)(x \ln 3 - 3)}{x^6}$ $g'(x) = \frac{3^x (x \ln 3 - 3)}{x^4}$	<p>6. <math>h(x) = \log_3(x^2)</math></p> $\frac{dh}{dx} = \frac{2x}{x^2} \cdot \frac{1}{\ln 3}$ <p style="text-align: right;">} log Rule</p> $\frac{dh}{dx} = \frac{2}{x \ln 3}$

7.  $k(x) = \log_4\left(\frac{x+2}{x-2}\right)$  Quotient Rule

$$k'(x) = \frac{(1)(x-2) - (x+2)(1)}{(x-2)^2} \cdot \frac{1}{\ln 4}$$

$$= \frac{x-2-x-2}{(x-2)^2} \cdot \frac{1}{\ln 4}$$

$$= \frac{-4}{(x-2)^2(x+2)} \cdot \frac{1}{\ln 4}$$

$$k'(x) = \frac{-4}{(x-2)(x+2)\ln 4}$$

8.  $f(x) = \log_2 \sqrt{2x-3} = \log_2 (2x-3)^{\frac{1}{2}}$

Chain Rule

$$f'(x) = \frac{\frac{1}{2}(2x-3)^{-\frac{1}{2}}(2)}{(2x-3)^{\frac{1}{2}}} \cdot \frac{1}{\ln 2}$$

$$f'(x) = \frac{1}{\ln 2(2x-3)}$$

**CALCULATOR**

9. Consider the functions  $f(x) = 5e^{\cos 2x}$  and  $g(x) = \log_3(5^{4x})$ . On the interval  $-1 \leq x \leq 1$ , for what value(s) of  $x$  will the two functions have parallel tangent lines? Show your work.

Chain

$$f'(x) = 5e^{\cos 2x} \cdot (-\sin(2x))(2)$$

$$f'(x) = -10 \sin(2x) \cdot e^{\cos 2x}$$

$$f' = g'$$

$$-10 \sin(2x) e^{\cos 2x} = 4 \log_3 5$$

**CALCULATOR**  $x = -0.111$

$$g(x) = 4x \cdot \log_3 5$$

$$g(x) = 4 \log_3 5 \cdot x$$

$$g'(x) = 4 \log_3 5 \text{ (Power Rule)}$$

When  $x \approx -0.111$ ,  $f(x)$  and  $g(x)$  will have parallel tangent lines on  $[-1, 1]$

**CALCULATOR**

10. For what value(s) of  $x$  on the interval  $-\pi < x < \pi$  will the graph of  $h(x) = 3^{-x} \cos(2x)$  have a tangent line whose slope is  $-1$ ?

Product Rule

Chain Rule

$$h'(x) = 3^{-x}(-1) \cdot \ln 3 \cos(2x) + 3^{-x}(-\sin(2x)) \cdot 2$$

$$h'(x) = -3^{-x} [\ln 3 \cdot \cos(2x) + 2 \sin(2x)]$$

When  $h'(x) = -1$

$$-1 = -3^{-x} [\ln 3 \cdot \cos(2x) + 2 \sin(2x)]$$

When  $x = -1.851, -0.033, \text{ and } 0.7$   
 $h(x)$  has a tangent line with slope of  $-1$ .

**CALCULATOR**

$$x \approx -3.387, -1.851, -0.033, 0.700$$

Not in Domain