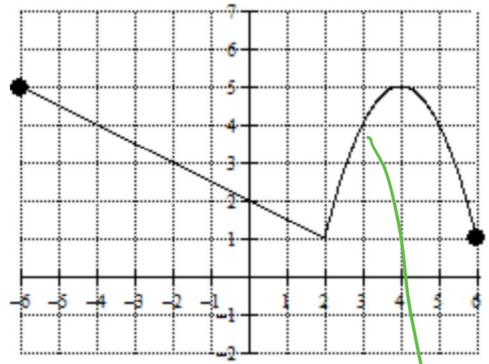
Name **Date** **Class**

Homework 3.7

Use the graph of $H(x)$, pictured to the right, to complete exercises 1 – 4.



1. The graph of $H(x)$ is continuous on its domain but not differentiable at all values on its domain. At what value on $-6 < x < 6$ is $H(x)$ not differentiable. Give a graphical reason for your answer.

   H(x) is not differentiable at $x=2$

   b/c the graph has a cusp at $x=2$.

2. (A) Write an equation of $H(x)$ and (B) show analytically that $H(x)$ is, in fact, continuous at the x – value that you identified in exercise 1. Show and explain your work.

(A)
$$H(x) = \begin{cases} -\frac{1}{2}x+2, & -6 \le x \le 2 \\ -(x-4)^2+5, & 2 < x \le 6 \end{cases}$$

(B)
I. $H(2)=1$ $\therefore$ $H(2)$ is defined

II. $\lim\limits_{x\to 2^-} H(x) = -\frac{1}{2}(2)+2 = 1$

$\lim\limits_{x\to 2^+} H(x) = -(2-4)^2+5 = 1$

$\therefore$ $\lim\limits_{x\to 2} H(x) = 1$ and therefore exists

III $H(2) = \lim\limits_{x\to 2} H(x) = 1$

$\therefore$ $H(x)$ is continuous at $x=2$ by the three part definition of continuity.

3. Show analytically that $H(x)$ is, in fact, not differentiable at the x – value that you identified in exercise 1. Show and explain your work.

$$H'(x) = \begin{cases} -\frac{1}{2}, & -6 \le x \le 2 \\ -2(x-4)(1), & 2 < x \le 6 \end{cases}$$

I. $H(x)$ is continuous at $x=2$

II. $\lim\limits_{x\to 2^-} H'(x) = -\frac{1}{2}$

$\lim\limits_{x\to 2^+} H'(x) = -2[(2)-4] = 4$

Since $\lim\limits_{x\to 2^-} H'(x) \ne \lim\limits_{x\to 2^+} H'(x)$,

$H(x)$ is not differentiable at $x=2$.

$$P(x) = [H(x)]^{1/2}$$

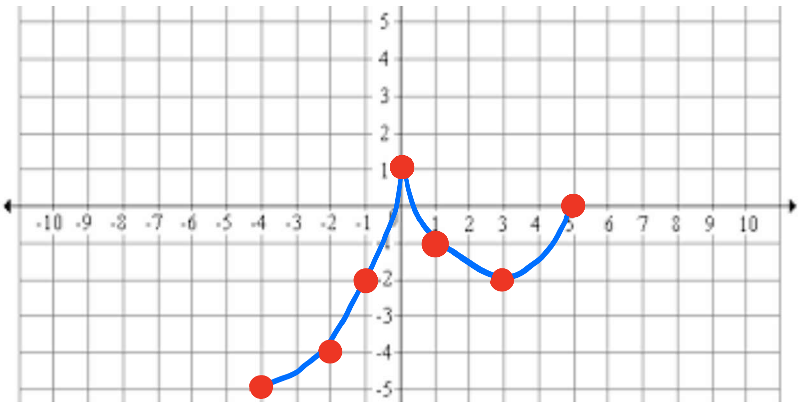
4. Given the graph of $H(x)$ pictured above, find the equation of the tangent line to the graph of $P(x) = \sqrt{H(x)}$ when $x = 3$.

POT: $(3,2)$	SOT: $m=\frac{1}{2}$	Tangent Line
$P(3)=\sqrt{H(3)}$ $=\sqrt{4}$ $P(3)=2$	$P'(x)=\frac{1}{2}[H(x)]^{-1/2}\cdot H'(x)$ $P'(x)=\frac{H'(x)}{2\sqrt{H(x)}}$ $P'(3)=\frac{H'(3)}{2\sqrt{H(3)}}$ $=\frac{-2[(3)-4]}{2\sqrt{4}}$ $=\frac{-2(-1)}{2\cdot 2}$ $P'(3)=\frac{1}{2}$	$y-2=\frac{1}{2}(x-3)$

A continuous function on the interval $-4 < x < 5$, $h(x)$, is described in the table below. Use the information to complete exercises 5 – 8.

x	-4	-2	-1	0	$-4 < x < 0$	1	3	$0 < x < 3$	$3 < x < 5$	5
h(x)	-5	-4	-2	1	Increasing & Concave Up	-1	-2	Decreasing & Concave Up	Increasing & Concave Up	0

5. Sketch a graph of $h(x)$.



6. Estimate the value of $h'(-2)$. Does this value support the claim that $h(x)$ is increasing on the interval $-4 < x < 0$? Give a reason for your answer.

$$h'(-2) \approx \frac{h(-4)-h(-1)}{(-4)-(-1)} = \frac{-5-(-2)}{-3} = \frac{-3}{-3} = 1$$

$h'(-2) \approx 1$ supports the claim that $h(x)$ is increasing on $(-4,0)$ because $h'(x) > 0$ on $(-4,0)$

7. There are three x – values in the domain of h at which $h(x)$ is not differentiable. What are these three values and give a reason for why $h(x)$ is not differentiable at these values.

- $H(x)$ is not differentiable at $x = -4, 0,$ or 5.
  - $H(x)$ is not continuous at $x = -4$ and $x = 5$.
  - $H(x)$ has a cusp at $x = 0$

8. On what interval(s) of x is $h'(x) > 0$? Give a reason for your answer.

$h' > 0$ on $(-4,0) \cup (3,5)$ because $h(x)$ is increasing on these intervals.

9. At what value(s) of x will the graph of $f(x) = 2e^{2x} - 3x$ have a tangent line whose slope is 1?

$\dot{f}(x) = 2e^{2x} \cdot (2) - 3$

$\dot{f}(x) = 4e^{2x} - 3$

$\boxed{\text{At } \dot{f}(x) = 1}$

$1 = 4e^{2x} - 3$

$4 = 4e^{2x}$

$1 = e^{2x}$

$e^0 = e^{2x}$

$0 = 2x$

Answer $\longrightarrow$ $0 = x$

10. The graph of $x - 2y = 9$ is parallel to the normal line to the graph of $f(x)$ when $x = 5$. What is the value of $f'(5)$? Justify your answer.

$x - 2y = 9$

$-2y = -x + 9$

$y = \frac{1}{2}x - \frac{9}{2}$

$SoN = \frac{1}{2}$

$SoT = -2$

The value of $f'(5) = -2$ because $f'(5)$ is the slope of the tangent line which is $\perp$ to the slope of the normal line at $x=5$. The slope of the normal line is the same as the slope of $x-2y=9$ because they are parallel with a slope of $\frac{1}{2}$.

11. Let f be defined by the function $f(x) = \begin{cases} 3 - x, & x < 1 \\ ax^2 + bx, & x \geq 1 \end{cases}$.

a. If the function is continuous at $x = 1$, what is the relationship between a and b? Explain your reasoning using limits.

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$

$3 - (1) = a(1)^2 + b(1)$

$2 = a + b$

b. Find the unique values of a and b that will make f both continuous and differentiable at $x = 1$. Show your analysis using limits.

$\lim_{x \to 1^-} f'(x) = \lim_{x \to 1^+} f'(x)$

$\lim_{x \to 1^-} -1 = \lim_{x \to 1^+} (2ax + b)$

$-1 = 2a(1) + b$

$-1 = 2a + b$

$$\boxed{\begin{aligned} 2 &= a + b \\ -1 &= 2a + b \end{aligned}} \Rightarrow \begin{aligned} 2 &= a + b \\ 1 &= -2a - b \\ \hline 3 &= -a \\ -3 &= a \end{aligned}$$

$2 = (-3) + b$

$5 = b$

$a = -3, b = 5$