

# Differentiating $a^{f(x)}$ and $\log_a f(x)$

Homework 4.1 Day 1

Find the derivative of each function.

#1)  $y = e^{2x^2}$

$$\begin{aligned} y' &= a^u \cdot \ln a \cdot u' \\ y' &= e^{2x^2} \cdot \ln e \cdot 4x \\ y' &= e^{2x^2} \cdot (1) \cdot 4x \\ y' &= 4x \cdot e^{2x^2} \end{aligned}$$

$a=e$	$u=2x^2$
	$u'=4x$

#2)  $y = e^{x^4}$

$$\begin{aligned} y' &= a^u \cdot \ln a \cdot u' \\ y' &= e^{x^4} \cdot \ln e \cdot 4x^3 \\ y' &= 4x^3 \cdot e^{x^4} \end{aligned}$$

$a=e$	$u=x^4$
	$u'=4x^3$

#3)  $f(x) = \ln(2x^3)$

$$\begin{aligned} f'(x) &= \frac{1}{u} \cdot \frac{1}{\ln a} \cdot u' \\ f'(x) &= \frac{1}{2x^3} \cdot \frac{1}{\ln e} \cdot (6x^2) \\ f'(x) &= \frac{6x^2}{2x^3} \cdot 1 \\ f'(x) &= \frac{3}{x} \end{aligned}$$

$a=e$	$u=2x^3$
	$u'=6x^2$

#4)  $f(x) = \ln(x - 5x^5)$

$$\begin{aligned} f'(x) &= \frac{1}{u} \cdot \frac{1}{\ln a} \cdot u' \\ f'(x) &= \frac{1}{x-5x^5} \cdot \frac{1}{\ln e} \cdot (1-25x^4) \\ f'(x) &= \frac{1-25x^4}{x-5x^5} \end{aligned}$$

$a=e$	$u=x-5x^5$
	$u'=1-25x^4$

#5)  $f(x) = \ln(x^6 + 5)$

$$\begin{aligned} f'(x) &= \frac{1}{u} \cdot \frac{1}{\ln a} \cdot u' \\ &= \frac{1}{x^6+5} \cdot \frac{1}{\ln e} (6x^5) \\ f'(x) &= \frac{6x^5}{x^6+5} \end{aligned}$$

#6)  $f(x) = 8^{\cos x}$

$$\begin{aligned} f'(x) &= a^u \cdot \ln a \cdot u' \\ f'(x) &= 8^{\cos x} \ln 8 \cdot (-\sin x) \\ f'(x) &= -\sin x \cdot \ln 8 \cdot 8^{\cos x} \end{aligned}$$

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#7)  $f(x) = \log_7(x^4)$

$$\begin{aligned} f'(x) &= \frac{1}{u} \cdot \frac{1}{\ln a} \cdot u' \\ &= \frac{1}{x^4} \cdot \frac{1}{\ln 7} \cdot 4x^3 \\ f'(x) &= \frac{4}{x^4 \ln 7} \end{aligned}$$

#8)  $y = \ln(\csc x + \tan x)$

$$\begin{aligned} y' &= \frac{1}{u} \cdot \frac{1}{\ln a} \cdot u' \\ &= \frac{1}{\csc x + \tan x} \cdot \frac{1}{\ln e} \cdot (-\csc x \cot x + \sec^2 x) \\ y' &= \frac{-\csc x \cot x + \sec^2 x}{\csc x + \tan x} \end{aligned}$$

#9)  $y = 2^{\tan x}$

$$\begin{aligned} y' &= a^u \cdot \ln a \cdot u' \\ y' &= 2^{\tan x} \ln 2 \cdot \sec^2 x \end{aligned}$$

#10)  $y = \ln 11^x$

$$y = x \underbrace{\ln 11}_{\text{constant}}$$

$$y' = x$$

Power Rule

#11)  $f(x) = e^{2x} - 2^{ex}$

$$\begin{aligned} f'(x) &= a^u \cdot \ln a \cdot u' - a^u \cdot \ln a \cdot u' \\ &= e^{2x} \cdot \ln e \cdot 2 \\ f'(x) &= 2e^{2x} - 2^{ex} \ln 2 \cdot e \end{aligned}$$

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Homework 4.1 Day 1

- #12) Find the values of  $x$  where the tangent to the graph of  $y = e^{2x}$  is parallel to  $12x - 2y = 6$

$$\begin{aligned}
 & f' \\
 & y' = a^u \cdot \ln a \cdot u' \\
 & y' = e^{2x} \cdot \ln e \cdot 2 \\
 & y' = 2e^{2x} \\
 & -2y = -12x + 6 \\
 & y = 6x - 3 \\
 & m = 6 \\
 & f' = m \\
 & 2e^{2x} = 6 \\
 & e^{2x} = 3 \\
 & \ln e^{2x} = \ln 3 \\
 & 2x = \ln 3 \\
 & x = \frac{\ln 3}{2}
 \end{aligned}$$

- #13) Find the values of  $x$  where the tangent to the graph of  $y = \frac{1}{e^{3x}}$  is parallel to  $5x + y = 109$

$$\begin{aligned}
 & y = e^{-3x} \\
 & y = -5x + 109 \\
 & m = -5
 \end{aligned}$$

$$\begin{aligned}
 & f' \\
 & y' = a^u \cdot \ln a \cdot u' \\
 & y' = e^{-3x} \cdot \ln e \cdot (-3) \\
 & y' = -3e^{-3x} \\
 & -3e^{-3x} = -5 \\
 & e^{-3x} = \frac{5}{3} \\
 & \ln e^{-3x} = \ln \frac{5}{3} \\
 & -3x = \ln \frac{5}{3} \\
 & x = -\frac{1}{3} \ln \frac{5}{3}
 \end{aligned}$$

- #14) At what coordinate point(s) is the tangent line of  $f(x) = \ln(x^3)$  parallel to  $y = 7 + 2x$ .  $\Rightarrow m=2$

$$\begin{aligned}
 & \textcircled{1} \quad \frac{\text{FIND } f'}{f'(x)} = \frac{1}{u} \cdot \frac{1}{\ln a} \cdot u' \\
 & = \frac{1}{x^3} \cdot \frac{1}{\ln e} \cdot 3x^2 \\
 & f'(x) = \frac{3}{x} \\
 & \textcircled{2} \quad \text{Set slope of line} = f' \\
 & 2 = \frac{3}{x} \\
 & 2x = 3 \\
 & x = \frac{3}{2} \\
 & \textcircled{3} \quad \frac{\text{Find } y\text{-value of point}}{f(x)} = \ln(x^3) \\
 & = \ln\left(\left(\frac{3}{2}\right)^3\right) \\
 & f(x) = \ln\left(\frac{27}{8}\right)
 \end{aligned}$$

$$\left(\frac{3}{2}, \ln\left(\frac{27}{8}\right)\right)$$

- Name \_\_\_\_\_  
 #15) If  $f(x) = e^{x^2}$ , what is the equation of the tangent line at  $x = 1$ ?

Point $(1, e)$	Slope $m=2e$	Point Slope
$f(1) = e^{1^2}$ $= e^1$	$f'(x) = a^u \cdot \ln a \cdot u'$ $f'(x) = e^{x^2} \cdot \ln e \cdot 2x$ $f'(x) = 2x \cdot e^{x^2}$ slope at $x=1$ $f'(1) = 2(1)e^{(1)^2}$ $= 2e^1$ $f'(1) = 2e$	$y - y_1 = m(x - x_1)$ $y - e = 2e(x - 1)$ $y = 2ex - 2e + e$ $y = 2ex - e$

- #16)  $f(x) = \ln(x^2)$  on the interval  $-1 < x < e$ . On this interval, when will the average rate of change equal the instantaneous rate of change? (Hint: Apply the Mean Value Theorem)

$$\begin{aligned}
 \text{ARC} &= \frac{f(e) - f(-1)}{e - (-1)} \\
 &= \frac{\ln(e)^2 - \ln(-1)^2}{e + 1} \\
 &= \frac{2 \ln e - \ln 1}{e + 1} \\
 &= \frac{2 - 0}{e + 1} \\
 &= \frac{2}{e + 1}
 \end{aligned}$$

$$\text{ARC} = \frac{2}{e+1}$$

MVT

$$\begin{aligned}
 \text{ARC} &= f'(c) \\
 \frac{2}{e+1} &= \frac{2}{c} \\
 2c &= 2(e+1) \\
 c &= e+1
 \end{aligned}$$

When  $x = e+1$