Differentiating $a^{f(x)}$ and $\log_a f(x)$

Homework 4.1 Day 2 Name_____

#13) $f(x) = x^7 7^x$

$(7^x)' = a^u \cdot \ln a \cdot u'$

$\boxed{a=7 \quad u=x \quad u'=1}$

$(7^x)' = 7^x \cdot \ln 7 \cdot 1$

$f'(x) = (x^7)' \cdot 7^x + x^7 \cdot (7^x)'$

$= 7x^6 \cdot 7^x + x^7 \cdot 7^x \cdot \ln 7$

$f'(x) = x^6 7^x [7 + x \ln 7]$

#14) $f(x) = \cos(\ln(2x^2))$ Double Chain

$f'(x) = -\sin[\ln(2x^2)] \cdot [\ln(2x^2)]'$

$= -\sin[\ln(2x^2)] \cdot \frac{1}{u} \cdot \frac{1}{\ln a} \cdot u'$

$= -\sin[\ln(2x^2)] \frac{1}{2x^2} \frac{1}{\ln e} 4x$

$= -\frac{2}{x} \sin[\ln(2x^2)]$

Test Prep

1. $\frac{d}{dx}(\ln(3x)\, 5^{2x}) =$ Product Rule!

(A) $\frac{5^{2x}}{x} + 2\ln(5)\ln(3x)\, 5^{2x}$ (B) $\frac{5^{2x}}{3x} - 2x\ln(3x)\, 5^{2x}$ (C) $\frac{5^{2x}}{x} - \ln(5)\ln(3x)\, 5^{2x}$

(D) $\frac{5^{2x}}{3x} + 2\ln(3x) 5^{2x}$ (E) $\frac{5^{2x}}{x} + \ln(5)\ln(3x)\, 5^{2x}$

$= \ln(3x)' \cdot 5^{2x} + \ln(3x) \cdot (5^{2x})'$

$= \frac{1}{u} \cdot \frac{1}{\ln a} \cdot u' \cdot 5^{2x} + \ln(3x) \cdot a^u \cdot \ln a \cdot u'$

$= \frac{1}{3x} \frac{1}{\ln e} \cdot 3 \cdot 5^{2x} + \ln(3x) \cdot 5^{2x} \cdot \ln 5 \cdot 2$

$= \frac{5^{2x}}{x} + 2\ln 5 \ln(3x) \cdot 5^{2x}$

2. The position of a particle moving along the x-axis is given by $x(t) = e^{2t} - e^t$ for all $t \geq 0$. When the particle is at rest, the acceleration of the particle is

$v = 0$

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\ln\frac{1}{2}$ (D) 2 (E) 4

$v(t) = 2e^{2t} - e^t$

rest $\rightarrow 0 = e^t(2e^t - 1)$

$0 = e^t$ dne

$0 = 2e^t - 1$

$1 = 2e^t$

$\frac{1}{2} = e^t$

$\ln\frac{1}{2} = \ln e^t$

$\ln\frac{1}{2} = t$

$a(t) = 4e^{2t} - e^t$

$a(\ln\frac{1}{2}) = 4e^{2\ln\frac{1}{2}} - e^{\ln\frac{1}{2}}$

$= 4e^{\ln(\frac{1}{2})^2} - \frac{1}{2}$

$= 4e^{\ln\frac{1}{4}}$

$= 4 \cdot \frac{1}{4} - \frac{1}{2}$

$= 1 - \frac{1}{2}$

$a(\ln\frac{1}{2}) = \frac{1}{2}$

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3. What is the slope of the curve $y = 3^{\sin x} - 2$ at its first positive x-intercept?

(A) 0.683 (B) 1.643 (C) 1.705 (D) 1.805 (E) 2

$y_1 = 3^{\sin x} - 2$

1st positive x-int = .683

2nd CALC $\frac{dy}{dx}$ x = .683

answer 1.705

4. Let $f(x) = 2e^{3x}$ and $g(x) = 5x^3$. At what value of x do the graphs of f and g have parallel tangents?

(A) −0.445 (B) −0.366 (C) −0.344 (D) −0.251 (E) −0.165

$f' = 6e^{3x} = y_1$

$g' = 15x^2 = y_2$

2nd Calc intersect

$x = -0.366$

5. If $y = 3\cos\left(\frac{x}{3}\right)$ then $\frac{d^2y}{dx^2} =$

(A) $-3\cos\left(\frac{x}{3}\right)$ (B) $-3\sin\left(\frac{x}{3}\right)$ (C) $-\frac{1}{3}\cos\left(\frac{x}{3}\right)$ (D) $-\frac{1}{3}\sin\left(\frac{x}{3}\right)$ (E) $-\cos\left(\frac{x}{3}\right)$

$y = 3\cos\left(\frac{x}{3}\right)$

$y' = -3\sin\left(\frac{x}{3}\right)\cdot\left(\frac{1}{3}x\right)'$

$y' = -3\sin\left(\frac{x}{3}\right)\cdot\left(\frac{1}{3}\right)$

$y' = -\sin\left(\frac{x}{3}\right)$

$y'' = -\cos\left(\frac{x}{3}\right)\cdot\left(\frac{1}{3}x\right)'$

$y'' = -\frac{1}{3}\cos\left(\frac{x}{3}\right)$

FREE RESPONSE **Your score: _____ out of 4**

2005 Form B AB3

A graphing calculator may be required. Use the space below the problem to show work and solutions.

1. A particle moves along the x-axis so that its velocity v at time t, for $0 \le t \le 5$, is given by

$$v(t) = \ln(t^2 - 3t + 3)$$

(a) Find the acceleration of the particle at time $t = 4$.

(b) Find all times t in the open interval $0 < t < 5$ at which the particle changes direction. During which time intervals, for $0 < t < 5$, does the particle travel to the left?

(a) $a(t) = [\ln(t^2 - 3t + 3)]'$ $\quad a = e \quad u = t^2 - 3t + 3$, $u' = 2t - 3$

$a(t) = \frac{1}{u} \cdot \frac{1}{\ln a} \cdot u'$

$= \frac{1}{t^2 - 3t + 3} \cdot \frac{1}{\ln e}(2t - 3)$

$a(t) = \frac{2t - 3}{t^2 - 3t + 3}$

$a(4) = \frac{2(4) - 3}{(4)^2 - 3(4) + 3}$

$= \frac{8 - 3}{16 - 12 + 3}$

$a(4) = \frac{5}{7}$ units/time^2

(b) When does $v(t) = 0$?

$\ln(t^2 - 3t + 3) = 0$

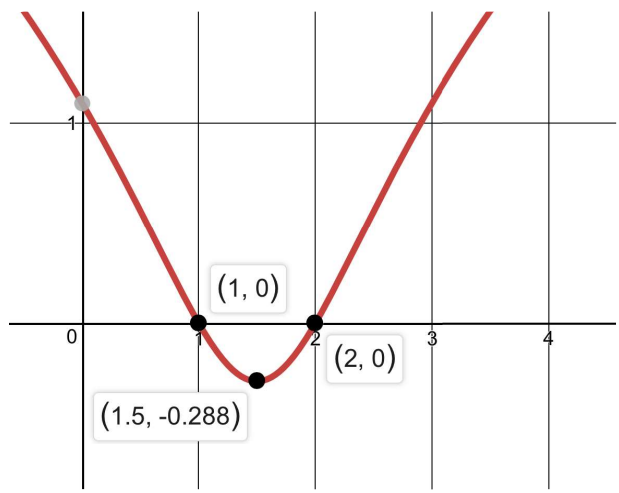
$e^0 = t^2 - 3t + 3$

$1 = t^2 - 3t + 3$

$0 = t^2 - 3t + 2$

$0 = (t - 1)(t - 2)$

$0 = t - 1$, $1 = t$ $\quad$ $0 = t - 2$, $2 = t$

The particle changes direction at time 1 and 2.

The particle travels left from time 1 to time 2.