

Differentiating $a^{f(x)}$ and $\log_a f(x)$

Homework 4.1 Day 2

#1) $y = e^{\cos(7x^3)}$

$$y' = a^u \cdot \ln a \cdot u'$$

$$y' = e^{\cos(7x^3)} \cdot \ln e \quad (\rightarrow x^2 \sin(7x^3))$$

$$y' = -21x^2 e^{\cos(7x^3)} \sin(7x^3)$$

$$\begin{aligned} a &= e & u &= \cos(7x^3) \\ u' &= -\sin(7x^3) \cdot (7x^3)' & u' &= -21x^2 \sin(7x^3) \end{aligned}$$

#2) $y = e^{\sin(5x^9)}$

$$\begin{aligned} a &= e & u &= \sin(5x^9) \\ u' &= \cos(5x^9) \cdot (5x^9)' & u' &= 45x^8 \cos(5x^9) \end{aligned}$$

$$y' = a^u \cdot \ln a \cdot u'$$

$$= e^{\sin(5x^9)} \cdot \ln e \cdot 45x^8 \cos(5x^9)$$

$$y' = 45x^8 \cos(5x^9) \cdot e^{\sin(5x^9)}$$

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#3) $f(x) = \ln(2x\sqrt{1+x})$

$$\begin{aligned} a &= e & u &= 2x\sqrt{1+x} \\ u' &= (2x)'(1+x)^{\frac{1}{2}} + 2x[(1+x)^{\frac{1}{2}}]' & u' &= 2(1+x)^{\frac{1}{2}} + 2x\left(\frac{1}{2}\right)(1+x)^{-\frac{1}{2}}(1+x)' \\ &= 2(1+x)^{\frac{1}{2}} + x(1+x)^{-\frac{1}{2}}(1) & & \text{Factor} \\ &= (1+x)^{\frac{1}{2}}[2(1+x) + x] & & \\ &= (1+x)^{\frac{1}{2}}(2+2x+x) & & \\ u' &= \frac{3x+2}{\sqrt{1+x}} & & \end{aligned}$$

$$f'(x) = \frac{1}{u} \cdot \frac{1}{\ln a} \cdot u'$$

$$= \frac{1}{2x\sqrt{1+x}} \cdot \frac{1}{\ln e} \cdot \frac{3x+2}{\sqrt{1+x}}$$

$$f'(x) = \frac{3x+2}{2x(1+x)}$$

#4) $f(x) = e^{x \sin x}$

$$\begin{aligned} a &= e & u &= x \sin x \\ u' &= x' \cdot \sin x + x \cdot (\sin x)' & u' &= 1 \cdot \sin x + x(-\cos x) \\ & & u' &= \sin x + x \cos x \end{aligned}$$

$$f'(x) = a^u \cdot \ln a \cdot u'$$

$$= e^{x \sin x} \ln e (\sin x + x \cos x)$$

$$f'(x) = (\sin x + x \cos x) e^{x \sin x}$$

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#5) $f(x) = \ln x \log x$
Product Rule!

$$f'(x) = (\ln x)' \cdot \log x + \ln x (\log x)'$$

$$f'(x) = \frac{1}{x} \cdot \frac{1}{\ln x} u' \cdot \log x + \ln x \cdot \frac{1}{u} \cdot \frac{1}{\ln x} u'$$

$$= \frac{1}{x} \cdot \frac{1}{\ln x} (1) \cdot \log x + \ln x \cdot \frac{1}{x} \cdot \frac{1}{\ln x} (1)$$

$$= \frac{\log x}{x} + \frac{\ln x}{x \ln 10}$$

$$= \frac{\log x}{x} + \frac{\log x}{x}$$

$$= \frac{\log x}{x} + \frac{\log x}{x}$$

$$= \frac{\log x}{x}$$

$$= \frac{2 \log x}{x}$$

$$= \frac{\log x}{x} \cdot \frac{\log 10}{\log e}$$

$$= \frac{\log x}{x \log 10}$$

$$\rightarrow \log 10 = 1$$

#6) $f(x) = x \ln(\sin 4x) - x^4$
Product Rule!

$$f'(x) = x' \cdot \ln(\sin 4x) + x [\ln(\sin 4x)]' - (x^4)'$$

$$f'(x) = 1 \cdot \ln(\sin 4x) + x \cdot \frac{1}{u} \cdot \frac{1}{\ln u} u' - 4x^3$$

$a=e$	$u=\sin 4x$
	$u'=\cos 4x \cdot (4x)'$
	$u'=4 \cos 4x$

$$f'(x) = \ln(\sin 4x) + x \cdot \frac{1}{\sin 4x} \cdot \frac{1}{\ln e} 4 \cos 4x - 4x^3$$

$$= \ln(\sin 4x) + 4x \frac{\cos 4x}{\sin 4x} - 4x^3$$

$$f'(x) = \ln(\sin 4x) + 4x \cot(4x) - 4x^3$$

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#7) $f(x) = e^{\pi x} - \ln(e^{\pi x})$

$$f(x) = e^{\pi x} - \pi x$$

$$f'(x) = a^u \cdot \ln a \cdot u' - (u \cdot x)'$$

$$= e^{\pi x} \cdot \ln e - \pi x$$

$$f'(x) = \pi e^{\pi x} - \pi x$$

#8) $y = e^{-5x} \cos 2x$
Product Rule!

$$y' = (-5x)' \cos 2x + e^{-5x} (\cos 2x)'$$

$$y' = a^u \cdot \ln a \cdot u' \cdot \cos 2x + e^{-5x} (-\sin 2x) \cdot (2x)'$$

$$y' = e^{-5x} \ln e (-5) \cos 2x - e^{-5x} \sin 2x \cdot 2$$

$$y' = -5e^{-5x} \cos 2x - 2e^{-5x} \sin 2x$$

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#9) $y = \frac{e^{\tan 3x}}{3x}$ Quotient Rule!

$$y' = \frac{(e^{\tan 3x})' \cdot 3x - e^{\tan 3x} (3x)'}{(3x)^2}$$

$$y' = \frac{a^u \ln a \cdot u' \cdot 3x - e^{\tan 3x} (3)}{9x^2}$$

$$\begin{aligned} a &= e & u &= \tan 3x \\ u' &= \sec^2(3x) \cdot (3x)' \\ u' &= \sec^2(3x) \cdot 3 \end{aligned}$$

$$y' = \frac{e^{\tan 3x} \ln e \cdot 3 \sec^2(3x) \cdot 3x - 3e^{\tan 3x}}{9x^2}$$

$$y' = \frac{9x \sec^2(3x) \cdot e^{\tan 3x} - 3e^{\tan 3x}}{9x^2}$$

$$y' = \frac{3e^{\tan 3x} (3x \sec^2(3x) - 1)}{9x^2}$$

$$y' = \frac{e^{\tan 3x} [3x \sec^2(3x) - 1]}{9x^2}$$

#10) $y = \ln\left(\frac{3x^2}{\sqrt{4-x^2}}\right)$ Simplify using log rules

$$\begin{aligned} y &= \ln 3x^2 - \ln \sqrt{4-x^2} & a &= e \\ a &= e & u &= 3x^2 & u &= e \\ u' &= 6x & u' &= \frac{1}{2}(4-x^2)^{\frac{1}{2}}(4x)' & u' &= 3 \\ u' &= \frac{1}{2}(4-x^2)^{\frac{1}{2}}(-2x) & u' &= -x(4-x^2)^{\frac{1}{2}} & u' &= 3 \end{aligned}$$

$$y' = \frac{1}{3x^2} \cdot \frac{1}{\ln e} (6x) - \frac{1}{\sqrt{4-x^2}} \cdot \frac{1}{\ln e} \cdot (-x(4-x^2)^{\frac{1}{2}})$$

$$y' = \frac{2 \cdot (4x^2)}{x(4-x^2)} - \frac{-x}{4-x^2} \cdot \frac{x}{x}$$

$$y' = \frac{8-2x^2+x^2}{x(4-x^2)}$$

$$y' = \frac{8-x^2}{x(4-x^2)}$$

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#11) $f(x) = \log\sqrt{10^{5x}}$

$$\begin{aligned} f(x) &= \frac{1}{2} \log 10^{5x} \\ f(x) &= \frac{5}{2} x \log 10 \\ f(x) &= \frac{5}{2} x \end{aligned}$$

$$f'(x) = \frac{5}{2}$$

#12) $y = \frac{\log_3 x}{e^{3x}}$ Quotient Rule

$$\begin{aligned} y' &= \frac{(\log_3 x)' e^{3x} - \log_3 x \cdot (e^{3x})'}{(e^{3x})^2} & a &= e & u &= 3x \\ a &= 3 & u &= x & u' &= 1 & a &= 3 & u &= 3 \\ u' &= 1 & u' &= 1 & u' &= 3 & u' &= 3 \\ y' &= \frac{\frac{1}{x} \ln a \cdot u' e^{3x} - \log_3 x \cdot a^u \cdot \ln a \cdot u'}{e^{6x}} & & & & & & & \end{aligned}$$

$$y' = \frac{\frac{1}{x} \cdot \frac{1}{\ln 3} \cdot 1 \cdot e^{3x} - \log_3 x \cdot e^{3x} \cdot \ln e \cdot 3}{e^{6x}}$$

$$y = \frac{\frac{1}{x \ln 3} - 3 \log_3 x}{e^{6x}}$$

$$y = \frac{\frac{1}{x \ln 3} - 3 \log_3 x}{e^{3x}}$$