Name Date Class

## Homework 4.1

For exercises 1 – 4, find $\frac{dy}{dx}$ by implicit differentiation.

1. $2x^3 - y^2 = 3y$

$$\frac{d}{dx}2x^3 - \frac{d}{dx}y^2 = \frac{d}{dx}3y$$
$$6x^2 - 2y\frac{dy}{dx} = 3\frac{dy}{dx}$$
$$6x^2 = 3\frac{dy}{dx} + 2y\frac{dy}{dx}$$
$$6x^2 = \frac{dy}{dx}(3+2y)$$
$$\frac{6x^2}{3+2y} = \frac{dy}{dx}$$

2. $x^3 - \underbrace{xy}_{\text{Product}} + y^2 = 4$

$$\frac{d}{dx}x^3 - \frac{d}{dx}(xy) + \frac{d}{dx}y^2 = \frac{d}{dx}4$$
$$3x^2 - \left[(1)\cdot y + x\cdot(1)\frac{dy}{dx}\right] + 2y\frac{dy}{dx} = 0$$
$$3x^2 - y - x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$
$$\frac{dy}{dx}(-x+2y) = y - 3x^2$$
$$\frac{dy}{dx} = \frac{y-3x^2}{2y-x}$$

3. $\underbrace{x^2 y}_{\text{Product}} + \underbrace{y^2 x}_{\text{Product}} = -2$

$$\frac{d}{dx}[x^2y] + \frac{d}{dx}[y^2x] = \frac{d}{dx}(-2)$$
$$2x\cdot y + x^2(1)\frac{dy}{dx} + 2y\frac{dy}{dx}x + y^2(1) = 0$$
$$x^2\frac{dy}{dx} + 2xy\frac{dy}{dx} = -y^2 - 2xy$$
$$\frac{dy}{dx}(x^2+2xy) = \frac{-y(y+2x)}{}$$
$$\frac{dy}{dx} = \frac{-y(y+2x)}{x(x+2y)}$$

4. $2\underbrace{\sin x \cos y}_{\text{Product}} = 1$

$$\frac{d}{dx}[2\sin x\cos y] = \frac{d}{dx}1$$
$$2\cos x\cdot\cos y + 2\sin x(-\sin y)\frac{dy}{dx} = 0$$
$$2\cos x\cos y = 2\sin x\sin y\frac{dy}{dx}$$
$$\frac{2\cos x\cos y}{2\sin x\sin y} = \frac{dy}{dx}$$
$$\frac{dy}{dx} = \cot x\cot y$$

5. $\boxed{\text{Find the coordinates of the points}}$ where the curve $4x^2 + y^2 - 8x + 4y + 4 = 0$ has a horizontal tangent.

SOT: $\frac{dy}{dx} = 0$

POT $(1,0), (1,-4)$

$$\frac{d}{dx}y^2 + \frac{d}{dx}4y = -\frac{d}{dx}4x^2 + \frac{d}{dx}8x - \frac{d}{dx}4$$

$$+2y\frac{dy}{dx} + 4\frac{dy}{dx} = 8 - 8x$$

$$\frac{dy}{dx}(2y+4) = 8 - 8x$$

$$\frac{dy}{dx} = \frac{2(4-4x)}{2(y+2)}$$

$$\frac{dy}{dx} = \frac{4-4x}{y+2}$$

$$0 = 4 - 4x$$
$$4x = 4$$
$$x = 1$$

$$y^2 + 4y = -4(1)^2 + 8(1) - 4$$
$$y^2 + 4y = -4 + 8 - 4$$
$$y^2 + 4y = 0$$
$$y(y+4) = 0$$
$$y = 0 \quad \Big\} \quad y + 4 = 0, \; y = -4$$

6. $\boxed{\text{Find the coordinates}}$ of the points where the curve $4x^2 + y^2 - 8x + 4y + 4 = 0$ has a vertical tangent.

Vertical tangent $\frac{dy}{dx} = \text{und}$

POT $(0,-2), (2,-2)$

$$\frac{dy}{dx} = \frac{4-4x}{y+2}$$

$$0 = y + 2$$
$$-2 = y$$

$$4x^2 + (-2)^2 - 8x + 4(-2) + 4 = 0$$
$$4x^2 + 4 - 8x - 8 + 4 = 0$$
$$4x^2 - 8x = 0$$
$$4x(x-2) = 0$$
$$4x = 0, \; x = 0 \quad \Big\} \quad x - 2 = 0, \; x = 2$$

$$(-2)^2 + 4(-2) = -4x^2 + 8x - 4$$
$$4 - 8 = -4x^2 + 8x - 4$$
$$\cancel{-4} = -4x^2 + 8x \cancel{-4}$$
$$0 = -4x(x-2)$$
$$0 = -4x, \; 0 = x \quad \Big\} \quad 0 = x - 2, \; 2 = x$$

7. Find (a) the equation of the tangent line and (b) the equation of the normal line drawn to the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 5$ at the point (8, 1).

SOT: $\frac{1}{2}$ SON: $-2$

$$\frac{d}{dx}x^{2/3} + \frac{d}{dx}y^{2/3} = \frac{d}{dx}5$$

$$\frac{2}{3}x^{-\frac{1}{3}} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$$

$$\frac{2}{3\sqrt[3]{x}} + \frac{2}{3\sqrt[3]{y}}\frac{dy}{dx} = 0$$

$$\frac{2}{3\sqrt[3]{y}}\frac{dy}{dx} = \frac{-2}{3\sqrt[3]{x}}$$

$$\frac{dy}{dx} = \frac{-2}{3\sqrt[3]{x}} \cdot \frac{3\sqrt[3]{y}}{2}$$

$$\frac{dy}{dx} = -\sqrt[3]{\frac{y}{x}}$$

$$\left.\frac{dy}{dx}\right|_{(8,1)} = -\sqrt[3]{\frac{1}{8}} = -\frac{1}{2}$$

Tangent Line

$$y - 1 = -\frac{1}{2}(x - 8)$$

Normal Line

$$y - 1 = +2(x - 8)$$

8. Find $\frac{d^2y}{dx^2}$ given the curve $y^2 = x^2 + 2x$.

$$\frac{d}{dx} y^2 = \frac{d}{dx} x^2 + \frac{d}{dx} 2x$$

$$2y \cdot \frac{dy}{dx} = 2x + 2$$

$$\frac{dy}{dx} = \frac{2(x+1)}{2y}$$

$$\frac{dy}{dx} = \frac{(x+1)}{y}$$

$$\frac{d^2y}{dx^2} = \frac{(1) \cdot y - (x+1)(1)\frac{dy}{dx}}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{y \cdot y - (x+1)\frac{(x+1)}{y} \cdot y}{y^2 \cdot y}$$

$$\frac{d^2y}{dx^2} = \frac{y^2 - (x+1)^2}{y^3}$$