Name Answer Key Date _____ Class _____

Day #29 Homework

1. The radius r and area A of a circle are related by the equation $A = \pi r^2$. Write an equation that relates $\frac{dA}{dt}$ to $\frac{dr}{dt}$.

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

2. The radius r and surface area S of a sphere are related by the equation $S = 4\pi r^2$. Write an equation that relates $\frac{dS}{dt}$ to $\frac{dr}{dt}$.

$$S = 4\pi r^2$$

$$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$$

The radius r, height h, and volume V of a right circular cylinder are related by the equation $V = \pi r^2 h$. Use this relationship to answer questions 3 – 5.

3. How is $\frac{dV}{dt}$ related to $\frac{dr}{dt}$ if h is constant?

$$V = \pi h r^2$$

$$\frac{dV}{dt} = 2\pi h r \frac{dr}{dt}$$

4. How is $\frac{dV}{dt}$ related to $\frac{dh}{dt}$ if r is constant?

$$V = \pi r^2 \cdot h$$

$$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$$

5. How is $\frac{dV}{dt}$ related to $\frac{dr}{dt}$ and $\frac{dh}{dt}$ if neither r nor h is constant?

$$V = \pi r^2 \cdot h$$

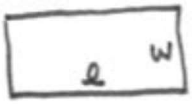
$$\frac{dV}{dt} = 2\pi r \frac{dr}{dt} \cdot h + \pi r^2 \cdot 1 \frac{dh}{dt}$$

$$\frac{dV}{dt} = 2\pi r h \frac{dr}{dt} + \pi r^2 \frac{dh}{dt}$$

Use the following information to solve problems 7 – 9. $-\frac{dl}{dt}$

$\frac{dw}{dt}$ The length of a rectangle is decreasing at the rate of 2 cm/sec while the width is increasing at the rate of 2 cm/sec. When $l = 12$ cm and $w = 5$ cm, find each of the rates of change of each quantity indicated below.

7. Area



$A = lw$

$\frac{dA}{dt} = 1 \cdot w \frac{dl}{dt} + l \cdot 1 \frac{dw}{dt}$

$\frac{dA}{dt} = 5(-2) + 12(2)$

$\frac{dA}{dt} = -10 + 24$

$\frac{dA}{dt} = \boxed{14 \text{ cm}^2/\text{sec}}$

8. Perimeter

$P = 2l + 2w$

$\frac{dP}{dt} = 2\frac{dl}{dt} + 2\frac{dw}{dt}$

$\frac{dP}{dt} = 2(-2) + 2(2)$

$\frac{dP}{dt} = \boxed{0 \text{ cm}/\text{sec}}$

9. Length of a diagonal of the rectangle

$12^2 + 5^2 = x^2$

$169 = x^2$

$x = 13$

144
25
169

$l^2 + w^2 = x^2$

$2l\frac{dl}{dt} + 2w\frac{dw}{dt} = 2x\frac{dx}{dt}$

$2(12)(-2) + 2(5)(2) = 2(13)\frac{dx}{dt}$

$-48 + 20 = 26\frac{dx}{dt}$

$\frac{-28}{26} = \frac{dx}{dt}$

$\frac{dx}{dt} = \boxed{-\frac{14}{13} \text{ cm}/\text{sec}}$

A spherical balloon is inflated with helium at the rate of 100π ft³/min $\frac{dV}{dt}$

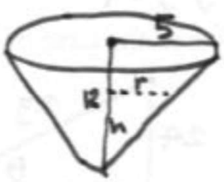
10. How fast is the balloon's radius increasing $\frac{dr}{dt}$ when the radius is 5 feet? r	11. How fast is the surface area increasing when the radius is 5 feet?
$V = \frac{4}{3}\pi r^3$ $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ $100\pi = 4\pi(5)^2 \frac{dr}{dt}$ $100\pi = 100\pi \frac{dr}{dt}$ $\frac{dr}{dt} = \boxed{1 \text{ ft per min}}$	$A = 4\pi r^2$ $\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$ $\frac{dA}{dt} = 8\pi(5)(1)$ $\frac{dA}{dt} = \boxed{40\pi \text{ ft}^2/\text{min}}$

12. A conical tank with the vertex down is 10 feet across the top and 12 feet deep. If water is flowing into the tank at a rate of 10 cubic feet per minute, find the rate of change of the depth of the water when the water is 8 feet deep. dV/dt $h=8$ $\frac{dh}{dt}$

$\frac{r}{h} = \frac{5}{12}$

$5h = 12r$

$r = \frac{5}{12}h$



$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{5}{12}h\right)^2 h = \frac{1}{3}\pi \frac{25}{144} h^3 = \frac{25}{432}\pi h^3$$

$$\frac{dV}{dt} = \frac{75}{432}\pi h^2 \frac{dh}{dt}$$

$$10 = \frac{75}{432}\pi(8)^2 \frac{dh}{dt}$$

$$10 = \frac{100\pi}{9} \frac{dh}{dt}$$

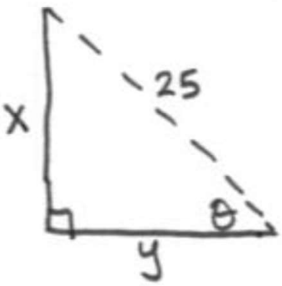
$$\frac{dh}{dt} = \frac{9}{100\pi} \cdot 10 = \boxed{\frac{9}{10\pi} \text{ feet per minute}}$$

A ladder is 25 feet long and is leaning against the wall of a house. The base of the ladder is pulled away from the wall at a rate of 2 feet per second. Use this information to complete exercises 13 – 15.

dy/dt

13. How fast is the top of the ladder moving down the wall when its base is 15 feet from the wall?

$\frac{dx}{dt}$ $\quad y$



$x^2 + y^2 = 25$

$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$

$2(20) \frac{dx}{dt} + 2(15)(2) = 0$

$40 \frac{dx}{dt} = 60$

$\frac{dx}{dt} = \boxed{-\frac{3}{2} \text{ feet/sec}}$

$x^2 + 15^2 = 25^2$

$x = 20$

* 14. Consider the triangle formed by the side of the house, the ladder and the ground. Find the rate at which the area of the triangle is changing when the ladder is 9 feet from the wall.

$A = \frac{1}{2} x y$

$\frac{dA}{dt} = \frac{1}{2} \frac{dx}{dt} \cdot y + \frac{1}{2} x \frac{dy}{dt}$

$\frac{dA}{dt} = \frac{1}{2}\left(\frac{-18}{\sqrt{544}}\right)(9) + \frac{1}{2}(\sqrt{544})(2)$

$\frac{dA}{dt} = \boxed{19.851 \text{ feet}^2 \text{ per sec}}$

$2x \frac{dx}{dt} = -2y \frac{dy}{dt}$

$2(\sqrt{544}) \frac{dx}{dt} = -2(9)(2)$

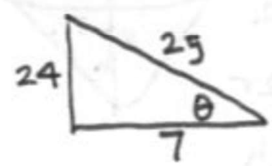
$\frac{dx}{dt} = \frac{-18}{\sqrt{544}}$

$x^2 + 9^2 = 25^2$

$x^2 = 544$

$x = \sqrt{544}$

15. Find the rate at which the angle between the ladder and the ground is changing when the base of the ladder is 7 feet from the wall.



$\cos\theta = \frac{y}{25}$

$\cos\theta = \frac{1}{25} y$

$-\sin\theta \frac{d\theta}{dt} = \frac{1}{25} \frac{dy}{dt}$

$-\frac{24}{25} \frac{d\theta}{dt} = \frac{1}{25}(2)$

$\frac{d\theta}{dt} = \frac{2}{25} \cdot -\frac{25}{24} = \boxed{-\frac{1}{12} \text{ radians per second}}$

$-\frac{dV}{dt}$ $\qquad \frac{r}{h} = \frac{45}{6} = \frac{15}{2} \qquad 2r = 15h$

$r = \frac{15}{2}h$

Water is flowing at a rate of 50 cubic meters per minute from a concrete conical reservoir. The radius of the reservoir is 45 m and the height is 6 m. Use this information to complete exercises 16 and 17.

r $\qquad$ h

16. How fast is the water level falling when the water is 5 meters deep?

h

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{15}{2}h\right)^2 h$

$V = \frac{1}{3}\pi\left(\frac{225}{4}h^2\right)h$

$V = \frac{75}{4}\pi h^3$

$\frac{dV}{dt} = \frac{225}{4}\pi h^2 \frac{dh}{dt}$

$-50 = \frac{225}{4}\pi (5)^2 \frac{dh}{dt}$

$-8 = 225\pi \frac{dh}{dt}$

$\frac{dh}{dt} = \boxed{-\frac{8}{225\pi} \text{ meters per minute}}$

17. How fast is the radius of the water's surface changing when the water is 5 meters deep?

$2r = 15h$

$h = \frac{2}{15}r$

$V = \frac{1}{3}\pi r^2 \cdot \frac{2}{15}r$

$V = \frac{2}{45}\pi r^3$

$2r = 15(5)$

$2r = 75$

$r = 75/2$

$\frac{dV}{dt} = \frac{2}{15}\pi r^2 \frac{dr}{dt}$

$-50 = \frac{2}{15}\pi\left(\frac{75}{2}\right)^2 \frac{dr}{dt}$

$-50 = \frac{375}{2}\pi \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{-100}{375\pi} = \boxed{-\frac{4}{15\pi} \text{ meters per minute}}$

18. A man 6 feet tall walks at a rate of 5 feet per second toward a street light that is 16 feet tall. At what rate is the length of his shadow changing when he is 10 feet from the base of the light?

dy/dt $\qquad$ dx/dt $\qquad$ y

$\frac{x}{x+y} = \frac{6}{16}$

$16x = 6x + 6y$

$10x = 6y$

$10x = 6y$

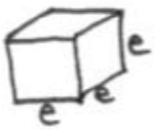
$10\frac{dx}{dt} = 6\frac{dy}{dt}$

$10\frac{dx}{dt} = 6(-5)$

$\frac{dx}{dt} = \boxed{-3 \text{ feet per second}}$

dV/dt $\quad$ $\frac{de}{dt}$

19. If the volume of a cube is increasing at a rate of 24 in^3/min and each edge is increasing at 2 in/min, what is the length of each edge of the cube?



$$V = e^3$$

$$\frac{dV}{dt} = 3e^2\frac{de}{dt}$$

$$24 = 3e^2(2)$$

$$4 = e^2$$

$$e = \boxed{2\text{ inches}}$$

dV/dt

20. If the volume of a cube is increasing at a rate of 24 in^3/min and the surface area is increasing at 12 in^2/min, what is the length of each edge of the cube?

$\frac{dA}{dt}$

$$V = e^3 \qquad A = 6e^2$$

$$\frac{dV}{dt} = 3e^2\frac{de}{dt} \qquad \frac{dA}{dt} = 12e\frac{de}{dt}$$

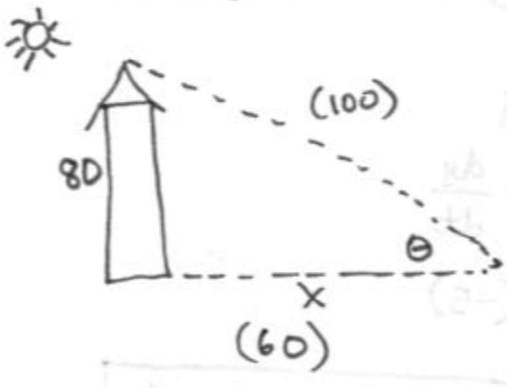
$$\frac{24}{3e^2} = \frac{de}{dt} \qquad \frac{12}{12e} = \frac{de}{dt}$$

$$\frac{24}{3e^2} = \frac{1}{e} \qquad 3e^2 - 24e = 0$$

$$3e^2 = 24e \nearrow 3e(e-8) = 0$$

$$\cancel{e=0} \quad \boxed{e = 8\text{ inches}}$$

21. On a morning when the sun will pass directly overhead, the shadow of an 80—foot building on level ground is 60 feet long. At the moment in question, the angle θ the sun makes with the ground is increasing at the rate of 0.27 radians per minute. At what rate is the length of the shadow decreasing?



$$\tan\theta = \frac{80}{x}$$

$$\tan\theta = 80x^{-1}$$

$$\sec^2\theta\,\frac{d\theta}{dt} = -80x^{-2}\frac{dx}{dt}$$

$$[\sec\theta]^2\frac{d\theta}{dt} = -\frac{80}{x^2}\frac{dx}{dt}$$

$$\left(\frac{5}{3}\right)^2(0.27) = -\frac{80}{(60)^2}\frac{dx}{dt}$$

$$\frac{dx}{dt} = \boxed{-33.75\text{ feet per minute}}$$