

## Homework 4.4

1. For the function  $h(x) = \frac{x^2 - 3x - 4}{x - 2}$ , determine the open intervals on which the given function is increasing or decreasing and the  $x$ -values of any relative extrema. Show your analysis and explain your reasoning.

! VA @  $x=2$  !

CV:  $x=2$

$$h'(x) = \frac{(2x-3)(x-2) - (x^2-3x-4)(1)}{(x-2)^2}$$

$$h'(x) = \frac{2x^2 - 3x - 4x + 6 - x^2 + 3x + 4}{(x-2)^2}$$

$$0 = \frac{x^2 - 4x + 10}{(x-2)^2}$$

ZON

$$\begin{aligned} x^2 - 4x + 10 &= 0 \\ \text{Disc} &= b^2 - 4ac \\ &= (-4)^2 - 4(1)(10) \\ &= 16 - 40 \\ \text{Disc} &= \text{neg} \\ h'(x) &\neq 0 \end{aligned}$$

ZOD

$$\begin{aligned} (x-2)^2 &= 0 \\ x-2 &= 0 \\ x &= 2 \end{aligned}$$

$h'(x)$  is undefined at  $x=2$

$$h'(x) = \frac{x^2 - 4x + 10}{(x-2)^2}$$

POS | POS  
 $x=0$  |  $x=1000$

increasing | increasing  
VA

- $h(x)$  is increasing on  $(-\infty, 2) \cup (2, \infty)$  b/c  $h' > 0$  on these intervals.
- $h(x)$  is never decreasing b/c  $h' \geq 0$ .
- $h(x)$  has no extrema b/c  $h'$  never changes sign.

2. If  $F'(x) = (x-1)^2(x-2)(x-4)$ , where is the graph of  $F(x)$  increasing, decreasing, and/or reaching a relative maximum or minimum? Show your work and justify your reasoning.

CV:  $x=1, 2, 4$

POS | POS | neg | POS  
 $x=0$  |  $x=1.5$  |  $x=3$  |  $x=5$

①      ③      ④

increasing | increasing | decreasing | increasing  
MAX      MIN

•  $F(x)$  has a relative min at  $x=4$  b/c  $F'(x)$  changes from  $-$  to  $+$  at  $x=4$

•  $F(x)$  has a relative max at  $x=2$  b/c  $F'(x)$  changes from  $+$  to  $-$  at  $x=2$ .

•  $F(x)$  is increasing on  $(-\infty, 1) \cup (1, 2) \cup (4, \infty)$  b/c  $F' > 0$  on these intervals.

b/c  $F' < 0$  on these intervals.

3. If  $h(x)$  is a twice differentiable function such that  $h(x) < 0$  for all values of  $x$  then at what value(s)

does the graph of  $g(x)$  have a relative maximum if  $g'(x) = (9-x^2)h(x)$ ?

CV:  $x=-3, 3$

$$\begin{cases} 0 = 9 - x^2 & h(x) \neq 0 \text{ b/c } h(x) < 0 \\ x^2 = 9 \\ x = \pm 3 \end{cases}$$

$g'(x) = 0 @ x = -3, 3$

$g'(x) \neq \text{und}$

POS | NEG | POS  
 $x=-4$  |  $x=0$  |  $x=4$

②      ③

increasing | decreasing | increasing  
MAX      MIN

•  $g(x)$  has a relative max at  $x=-3$  b/c  $g'(x)$  changes from  $+$  to  $-$  at  $x=-3$

For exercises 4 – 5, identify the intervals where the function,  $g(x)$ , is concave up and concave down. Also, identify the  $x$  – values of any points of inflection. Show your work and justify your reasoning.

Ch 4.1

4.  $g'(x) = \sqrt{8x - x^2}$

CV

$$g''(x) = \frac{1}{2} (8x - x^2)^{-\frac{1}{2}} (8 - 2x)$$

$$0 = \frac{4-x}{\sqrt{8x-x^2}}$$

$$\text{ZOM: } x=4$$

$$\text{ZOD: } x=0, 8$$

$$4-x=0$$

$$4=x$$

$$g''(4)=0 \text{ at } x=4$$

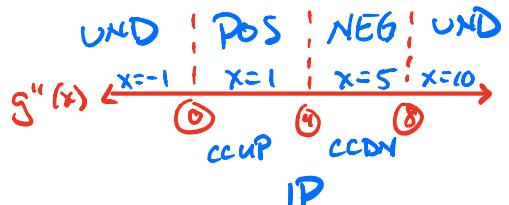
$$\sqrt{8x-x^2}=0$$

$$x(8-x)=0$$

$$x=0 \quad | \quad 8-x=0 \\ x=8$$

$$g''(x) = \text{undefined at } x=0, x=8$$

$$g''(x) = \frac{4-x}{\sqrt{x(8-x)}}$$



- $g(x)$  is concave up on  $(0, 4)$  b/c  $g'' > 0$  on  $(0, 4)$

- $g(x)$  is concave down on  $(4, 8)$  b/c  $g'' < 0$  on  $(4, 8)$

- $g(x)$  has an inflection point at  $x=4$  b/c  $g''$  changes signs at  $x=4$ .

5.  $g(x) = xe^{2x}$

$$g'(x) = 1 \cdot e^{2x} + x \cdot e^{2x} \cdot 2$$

$$g'(x) = e^{2x} + 2xe^{2x}$$

$$g''(x) = 2e^{2x} + 2 \cdot e^{2x} + 2x \cdot e^{2x} \cdot 2$$

$$g''(x) = 4e^{2x} + 4xe^{2x}$$

$$g''(x) = 4e^{2x}(1+x)$$

$$\text{CV: } x=-1$$

$$g''(x) = 4e^{2x}(1+x)$$

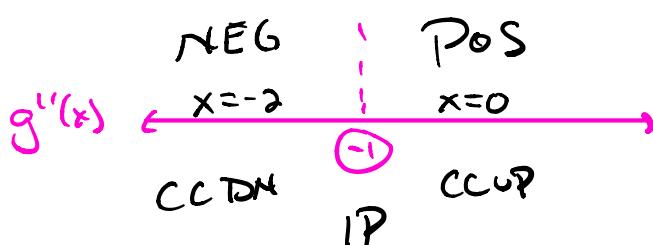
$$0 = 4e^{2x}(1+x)$$

$$0 = 4e^{2x} \quad | \quad 0 = 1+x$$

$$\text{No solution} \quad | \quad -1 = x$$

$$g''(x) = 0 \text{ at } x=-1$$

$$g''(x) \neq \text{und}$$



- $g(x)$  is concave up on  $(-1, \infty)$  b/c  $g'' > 0$  on  $(-1, \infty)$

- $g(x)$  is concave down on  $(-\infty, -1)$  b/c  $g'' < 0$  on  $(-\infty, -1)$

- $g(x)$  has an inflection point at  $x=-1$  b/c  $g''$  changes signs at  $x=-1$

For exercises 6 and 7, use the Second Derivative Test to find the local extrema for the given function. Show your analysis and justify your reasoning.

6.  $g(x) = 3x - x^3 + 5$

CV:  $x = -1, 1$

$g'(x) = 3 - 3x^2$	$0 = 3(1 - x^2)$
ZoN	ZoD
$0 = 1 - x^2$	$g'(x) \neq \text{cond}$
$x^2 = 1$	
$x = \pm 1$	
$g'(x) = 0 \text{ at } x = \pm 1$	

### 2nd Derivative Test

$$g''(x) = -6x$$

$$g''(-1) = -6(-1) > 0, g(x) \text{ is CC UP} \therefore \text{MIN at } x = -1$$

$$g''(1) = -6(1) < 0, g(x) \text{ is CC DN} \therefore \text{MAX at } x = 1$$

- $g(x)$  has a local min at  $x = -1$  b/c  $g''(-1) > 0$
- $g(x)$  has a local max at  $x = 1$  b/c  $g''(1) < 0$

7.  $h(x) = x^3 + 3x^2 - 2$

CV:  $x = -2, 0$

$h'(x) = 3x^2 + 6x$

$0 = 3x(x+2)$

ZoN	ZoD
$0 = 3x \begin{cases} 0 = x+2 \\ x = 0 \end{cases}$	$h'(x) \neq \text{cond}$
$x = -2$	

$h'(x) = 0 \text{ at } x = 0, -2$

### 2nd Derivative Test

$$h''(x) = 6x+6$$

$$h''(x) = 6(x+1)$$

$$h''(-2) = 6(-2+1) < 0, h \text{ is CC DN} \therefore \text{MAX at } x = -2$$

$$h''(0) = 6(0+1) > 0, h \text{ is CC UP} \therefore \text{MIN at } x = 0$$

- $h(x)$  has a local min at  $x = 0$  b/c  $h''(0) > 0$
- $h(x)$  has a local max at  $x = -2$  b/c  $h''(-2) < 0$

