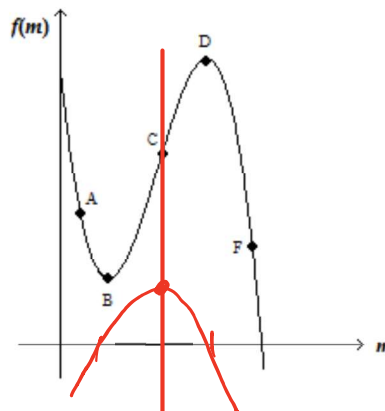


Homework 4.6

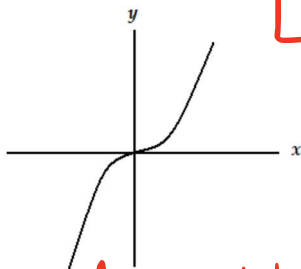
1. Pictured below is a function, $f(m)$. Complete the chart below indicating the sign (+ or - or 0) for $f(m)$, $f'(m)$ and $f''(m)$ at each of the indicated points.

Point	$f(m)$	$f'(m)$	$f''(m)$
A	+	-	+
B	+	0	+
C	+	+	0
D	+	0	-
F	+	-	-



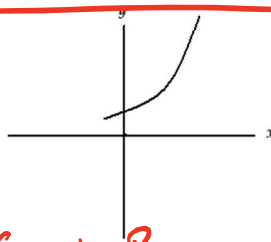
2. If, for all real numbers x , $f'(x) < 0$ and $f''(x) > 0$, which of the following curves could be part of the graph of $f(x)$? Explain your reasoning FOR EACH GRAPH.

Graph A



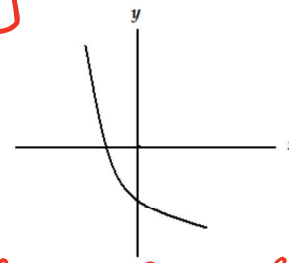
Graph A cannot be part of $f(x)$ b/c A is increasing on $(-\infty, \infty)$ and A is concave down $(-\infty, 0)$

Graph B



Graph B cannot be part of $f(x)$ b/c B is increasing on $(-\infty, \infty)$

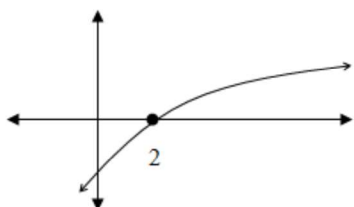
Graph C



Graph C can be part of $f(x)$ b/c C is decreasing and concave up on $(-\infty, \infty)$

If $f' < 0$, then f is decreasing
If $f'' > 0$, then f is concave up

3. The graph of a twice differentiable function is shown below. Order the values of $f(2)$, $f'(2)$ and $f''(2)$ in order from least to greatest. Explain your reasoning.

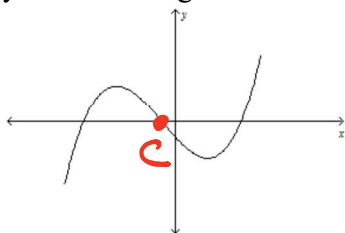


$$f''(2) < f(2) < f'(2)$$

$f''(2) < 0$ b/c $f(x)$ is concave down at $x=2$
 $f(2) = 0$ b/c 2 is a root
 $f'(2) > 0$ b/c $f(x)$ is increasing at $x=2$

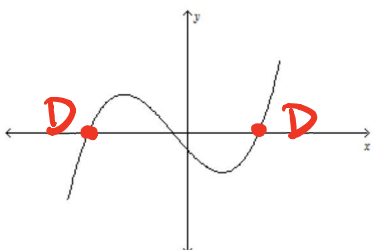
The graph of $f'(x)$, the derivative of $f(x)$ is shown in each of the following questions. Answer the questions 4 – 6 using this graph.

4. How many relative maximums does $f(x)$ have? Label these x values with the letter C. Explain your reasoning.



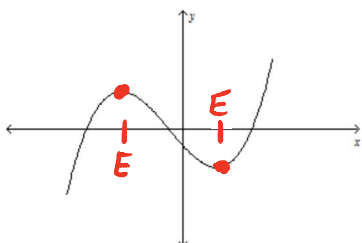
- $f(x)$ has one relative max b/c
 - $f'(x)$ changes from pos. to neg. in one location.

5. How many relative minimums does $f(x)$ have? Label these x values with the letter D. Explain your reasoning.



- $f(x)$ has 2 relative min b/c
 - $f'(x)$ changes from neg. to pos. in two locations.

6. How many points of inflection does the graph of $f(x)$ have? Label these x values with the letter E. Explain your reasoning.

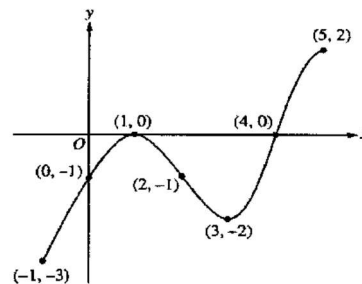


- $f(x)$ has 2 inflection points b/c
 - $f'(x)$ changes has two rel. extrema.

Pictured to the right is the graph of $f'(x)$. Use the graph to answer questions 7 – 13.

7. What are the value(s) of x where $f(x)$ has a relative maximum?
 Explain your reasoning.

- $f(x)$ does not have a relative max
 - b/c $f'(x)$ never changes from + to -.



8. What are the value(s) of x where $f(x)$ has a relative minimum?
 Explain your reasoning.

- $f(x)$ has a relative min at $x=4$
 - b/c $f'(x)$ changes from - to + at $x=4$.

9. On what interval(s) is the graph of $f(x)$ increasing? Explain your reasoning.

- $f(x)$ is increasing on $(4, 5)$
 - b/c $f'(x) > 0$ on $(4, 5)$

10. At what value(s) of x does the graph of $f(x)$ have a point of inflection? Explain your reasoning.

- $f(x)$ has IP at $x=1$ and $x=3$
 - b/c $f'(x)$ has relative extrema at $x=1$ and $x=3$

11. On what interval(s) is the graph of $f(x)$ concave up or concave down? Explain your reasoning.

$f(x)$ is concave down on $(1,3)$ b/c $f' < 0$ on $(1,3)$
 $f(x)$ is concave up on $(-1,1) \cup (3,5)$ b/c $f' > 0$ on these intervals.

12. If $f(2) = 4$, what is the equation of the normal line to the graph of $f(x)$ when $x = 2$?

PoN SoT SoN Normal Line
 $(2,4)$ $f'(2) = -1$ $m = 1$ $y - 4 = 1(x - 2)$

13. If $f(2) = 4$, what is the tangent line approximation of $f(1.9)$? Is this an over or under approximation of $f(1.9)$? Explain your reasoning.

Tangent
 $y - 4 = -1(x - 2)$

$y - 4 = -1(1.9 - 2)$
 $y - 4 = -1(-0.1)$
 $y = 4 + 0.1$
 $y = 4.1$

$f(1.9) \approx 4.1$
 $f(1.9) \approx 4.1$ is an over approximation because $f' < 0$, so f is decreasing, and $f'' < 0$, so f is concave down.

$(1.9, f' < 0)$
 $(1.9, f'' < 0)$

A function, F , is continuous on its domain of $[-2, 4]$. Additionally, $F(-2) = 5$, $F(4) = 1$ with F' and F'' have the properties shown in the table below. Use this information to answer questions 14 – 17.

x	$-2 < x < 0$	$x = 0$	$0 < x < 2$	$x = 2$	$2 < x < 4$
$F'(x)$	Positive	Does not exist	Negative	0	Negative
$F''(x)$	Positive	Does not Exist	Positive	0	Negative

14. At what value(s) of x does F have relative extrema? Classify the extrema by type and give a reason for your answer.

F has relative max at $x=0$ because F' changes from $+$ to $-$ at $x=0$

15. At what value(s) of x does F have a point of inflection? Justify your answer.

F has a point of inflection at $x=2$ b/c F'' changes signs at $x=2$.

16. On what interval(s) is the graph of F increasing, decreasing, concave up or concave down? Justify your reasoning.

F is increasing on $(-2, 0)$ b/c $F' > 0$ on $(-2, 0)$

F is decreasing on $(0, 2) \cup (2, 4)$ b/c $F' < 0$ on $(0, 2) \cup (2, 4)$

F is concave up on $(-2, 0) \cup (0, 2)$ b/c $F'' > 0$ on $(0, 2) \cup (2, 4)$

F is concave down on $(2, 4)$ b/c $F'' < 0$ on $(-2, 0)$

17. Suppose the equation of the tangent line drawn to F at $x = 2$ were used to evaluate $F(1.6)$ and $F(2.4)$. Would the approximations be under or over approximations? Justify your answers.

$F(2)$ is an under approximation of $F(1.6)$ because

F is concave up and decreasing at $x = 1.6$

$F(2)$ is an over approximation of $F(2.4)$ because

F is concave down and decreasing at $x = 2.4$

