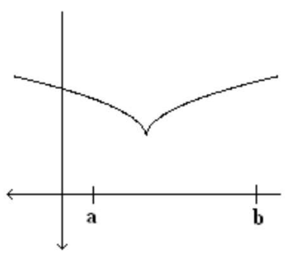
Name______________________________Date________________Class__________

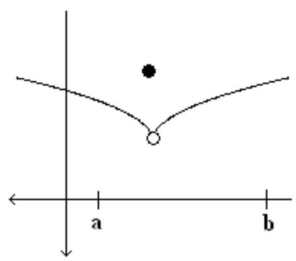
Homework 5.1

1. For which of the following functions is the Extreme Value Theorem NOT APPLICABLE on the interval [*a*, *b*]? Give a reason for your answer.

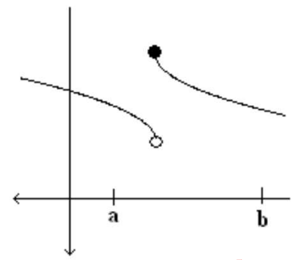
Graph I



Graph II



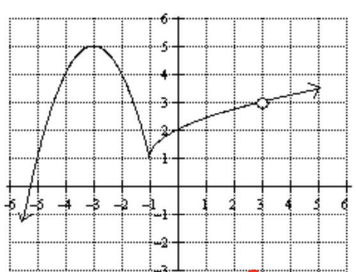
Graph III



Graph II and III are not continuous on [a,b]

∴ The E.V.T does not apply.

For exercises 2 – 4, determine the critical numbers for each of the functions below.

2.  $x = -3, -1, 3$ $f'(x) = 0$ at $x = -3$ $f'(x) = \text{und}$ at $x = -1, 3$	3. $g(x) = \ln(x^2 + 4)$ $g' = \frac{2x}{x^2+4}$ $g' = 0$: $2x = 0$, $x = 0$ $g' = \text{und}$: $x^2 + 4 = 0$, $x^2 = -4$, $x = \pm 2i$ (not real) ∴ $x = 0$	4. $h(x) = \sqrt[3]{x+3}$ $= (x+3)^{1/3}$ Chain $h' = \frac{1}{3}(x+3)^{-2/3}(1)$ $h' = \frac{1}{3(x+3)^{2/3}}$ $h' = 0$: $0 = 1$ No solution $h' = \text{und}$: $3(x+3)^{2/3} = 0$, $(x+3)^{2/3} = 0$, $x+3 = 0$, $x = -3$ ∴ $x = -3$

5. Given the function below, apply the Extreme Value Theorem to find the absolute extrema of $f(x)$ on the indicated interval. Point

CALC

$f(x) = \sin x \cdot \ln(x+1)$ on the interval [1, 6]

$f' = \cos x \cdot \ln(x+1) + \sin x \cdot \frac{1}{x+1}$

$f' = \cos x \cdot \ln(x+1) + \frac{\sin x}{x+1}$

$f' = 0$ (calc)

$x = 1.887$

$x = 4.810$

$f' = \text{und}$

$x + 1 = 0$, $x = -1$; $x + 1 \le 0$, $x \le -1$

not in Domain

CALC

$$\boxed{\begin{aligned} &EV: f(1) = 0.583 \\ &CV: f(1.887) = 1.008 \\ &CV: f(4.810) = -1.751 \\ &EV: f(6) = -0.544 \end{aligned}}$$

∴ ABSOLUTE MAX @ (1.887, 1.008)

ABSOLUTE MIN @ (4.810, −1.751)

Y

For exercises 6 – 9, determine the absolute extreme values on the given interval. You should do each of these independent from a calculator.

6. $f(x) = x^3 - 3x^2$ on the interval $[-1, 3]$

$f' = 3x^2 - 6x$

$f' = 0$: $3x(x-2) = 0$, $x = 0,\ x = 2$

f' = und: $\varnothing$

EV $f(-1) = (-1)^3 - 3(-1)^2 = -1 - 3 = -4$

CV $f(0) = (0)^3 - 3(0)^2 = 0$

CV $f(2) = (2)^3 - 3(2)^2 = 8 - 3(4) = 8 - 12 = -4$

EV $f(3) = (3)^3 - 3(3)^2 = 27 - 27 = 0$

$\therefore$ ABSOLUTE MAX: $y = 0$

ABSOLUTE MIN: $y = -4$

7. $g(x) = \sqrt[3]{x+2} = (x+2)^{1/3}$ on the interval $[-3, 6]$

$g' = \frac{1}{3}(x+2)^{-2/3}(1)$

$g' = \dfrac{1}{3(x+2)^{2/3}}$

$g' = 0$: no solution

g' = und: $3(x+2)^{2/3} = 0$, $(x+2)^{2/3} = 0$, $x + 2 = 0$, $x = -2$

EV: $g(-3) = \sqrt[3]{(-3)+2} = \sqrt[3]{-1} = -1$

CV: $g(-2) = \sqrt[3]{(-2)+2} = \sqrt[3]{0} = 0$

EV: $g(6) = \sqrt[3]{(6)+2} = \sqrt[3]{8} = 2$

$\therefore$ ABSOLUTE MAX: $y = 2$

ABSOLUTE MIN: $y = -1$

8. $h(x) = \frac{x}{x+2}$ on the interval $[-4, 0]$

$h(x)$ is not continuous on $[-4, 0]$ due to VA @ $x = -2$. $\therefore$ The E.V.T. does not apply for $h(x)$ on $[-4, 0]$

9. $f(x) = 3x^{\frac{2}{3}} - 2x$ on the interval $[-1, 1]$

$f' = 2x^{-1/3} - 2$

$f' = \dfrac{2}{x^{1/3}} - 2 \cdot \dfrac{x^{1/3}}{x^{1/3}}$

$f' = \dfrac{2 - 2x^{1/3}}{x^{1/3}}$

$f' = 0$: $2 - 2x^{1/3} = 0$, $2 = 2x^{1/3}$, $1 = x^{1/3}$, $1 = x$

f' = und: $x^{1/3} = 0$, $x = 0$

EV: $f(-1) = 3\sqrt[3]{(-1)^2} - 2(-1) = 3 + 2 = 5$

CV: $f(0) = 3\sqrt[3]{0^2} - 2(0) = 0 - 0 = 0$

CV/EV: $f(1) = 3\sqrt[3]{1^2} - 2(1) = 3 - 2 = 1$

$\therefore$ ABSOLUTE MAX: $y = 5$

ABSOLUTE MIN: $y = 0$