

Homework 5.2

For the exercises 1 – 5, determine whether Rolle's Theorem can be applied to the function on the indicated interval. If Rolle's Theorem can be applied, find all values of c that satisfy the theorem.

1. $f(x) = x^2 - 4x$ on the interval $0 \leq x \leq 4$

- a) $f(x)$ is continuous on $[0, 4]$
- b) $f(x)$ is differentiable on $(0, 4)$
- c) $f(0) = f(4) = 0$.

Rolle's Theorem applies.

$$\begin{aligned}f'(x) &= 2x - 4 \\f'(c) &= 0 \\2c - 4 &= 0 \\2c &= 4 \\c &= 2\end{aligned}$$

3. $f(x) = 4 - |x - 2|$ on the interval $-3 \leq x \leq 7$

- a) $f(x)$ is continuous on $[-3, 7]$
- b) $f(x)$ is not differentiable at $x=2$.

Rolle's Theorem does not apply.

5. $f(x) = \cos 2x$ on the interval $\frac{\pi}{3} \leq x \leq \frac{2\pi}{3}$

- a) $f(x)$ is continuous on $[\frac{\pi}{3}, \frac{2\pi}{3}]$
- b) $f(x)$ is differentiable on $(\frac{\pi}{3}, \frac{2\pi}{3})$
- c) $f(\frac{\pi}{3}) = f(\frac{2\pi}{3}) = -\frac{1}{2}$

Rolle's Theorem applies

$$\text{Graph: } \cos[2(\frac{\pi}{3})] = -\frac{1}{2}$$

$$\cos[2(\frac{2\pi}{3})] = \cos \frac{4\pi}{3} = -\frac{1}{2}$$

2. $f(x) = (x+4)^2(x-3)$ on the interval $-4 \leq x \leq 3$

- a) $f(x)$ is continuous on $[-4, 3]$
- b) $f(x)$ is differentiable on $(-4, 3)$
- c) $f(-4) = f(3) = 0$.

Rolle's Theorem applies.

$$\begin{aligned}f' &= 2(x+4)(1)(x-3) + (x+4)^2(1) \\f' &= (x+4)[2(x-3) + (x+4)] \\f' &= (x+4)[2x-6+x+4] \\f' &= (x+4)(3x-2) \\f'(c) &= 0 \\(c+4)(3c-2) &= 0 \\c = -4, c = \frac{2}{3} &\quad \text{Not in Domain} \quad \therefore c = \frac{2}{3}\end{aligned}$$

4. $f(x) = \sin x$ on the interval $0 \leq x \leq 2\pi$

- a) $f(x)$ is continuous on $[0, 2\pi]$
- b) $f(x)$ is differentiable on $(0, 2\pi)$
- c) $f(0) = f(2\pi) = 0$

Rolle's Theorem applies

$$\begin{aligned}f' &= \cos x \\f'(c) &= 0 \\\cos c &= 0 \\c = \frac{\pi}{2}, c = \frac{3\pi}{2} &\quad \text{X}\end{aligned}$$

$$f' = -\sin(2x) \cdot 2$$

$$f' = -2\sin(2x)$$

$$f'(c) = 0$$

$$-2\sin(2c) = 0$$

$$\sin(2c) = 0$$

$$2c = \pi, 2\pi$$

$$c = \frac{\pi}{2}, c = \pi \quad \text{X}$$

$$c = \frac{\pi}{2}$$

For exercises 6 – 9, determine whether the Mean Value Theorem can be applied to the function on the indicated interval. If the Mean Value Theorem can be applied, find all values of c that satisfy the theorem.

$$-1 < c < 1$$

6. $f(x) = x^3 - x^2 - 2x$ on $-1 \leq x \leq 1$

- a) $f(x)$ is continuous on $[-1, 1]$
 b) $f(x)$ is differentiable on $(-1, 1)$
- The Mean Value Theorem applies.

$$f'(c) = \frac{f(-1) - f(1)}{-1 - 1}$$

$$3c^2 - 2c - 2 = \frac{0 - (-2)}{-2}$$

$$= \frac{2}{-2}$$

$$3c^2 - 2c - 2 = -1$$

$$3c^2 - 2c - 1 = 0$$

$$\frac{(3c-3)(3c+1)}{3} = 0$$

$$(c-1)(3c+1) = 0$$

$$\cancel{c-1}, c = -\frac{1}{3} \quad \therefore c = -\frac{1}{3}$$

8. $f(x) = \frac{x+2}{x}$ on $\frac{1}{2} \leq x \leq 2$

- a) $f(x)$ is continuous on $[\frac{1}{2}, 2]$
 b) $f(x)$ is differentiable on $(\frac{1}{2}, 2)$
- The Mean Value Theorem applies.

$$f'(x) = \frac{1 \cdot x - (x+2)(1)}{x^2} = \frac{x - x - 2}{x^2} = \frac{-2}{x^2}$$

$$f'(c) = \frac{\frac{1}{2} - f(2)}{\frac{1}{2} - 2}$$

$$\frac{-2}{c^2} = \frac{\frac{5}{2}}{\frac{1}{2}} - \frac{4}{2}$$

$$-1.5$$

$$\frac{-2}{c^2} = \frac{5-2}{-3/2}$$

$$\frac{-2}{c^2} = -\frac{3}{3/2}$$

$$\frac{-2}{c^2} = -2$$

$$-2 = -2c^2$$

$$1 = c^2$$

$$\cancel{c \neq 1}, c = 1$$

7. $f(x) = \sqrt{x-3}$ on $3 \leq x \leq 7$

- a) $f(x)$ is continuous on $[3, 7]$
 b) $f(x)$ is differentiable on $(3, 7)$

The Mean Value Theorem applies.

$$f'(c) = \frac{f(3) - f(7)}{3 - 7}$$

$$\frac{1}{2}(c-3)^{-1/2} = \frac{0 - \sqrt{7-3}}{-4}$$

$$\frac{1}{2(c-3)^{1/2}} = \frac{-2}{-4}$$

$$\frac{1}{2(c-3)^{1/2}} = \frac{1}{2}$$

$$2 = 2(c-3)^{1/2}$$

$$1 = (c-3)^{1/2}$$

$$1 = c-3$$

$$\therefore c = 4$$

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9. $h(x) = 2 \cos x + \cos 2x$ on $0 \leq x \leq \pi$

- a) $f(x)$ is continuous on $[0, \pi]$
 b) $f(x)$ is differentiable on $(0, \pi)$
- The Mean Value Theorem applies.

$$h' = -2 \sin x - 2 \sin(2x)$$

$$f'(c) = \frac{h(0) - h(\pi)}{0 - \pi}$$

$$-2 \sin c - 2 \sin(2c) = \frac{[2 \cos 0 + \cos 2(0)] - [2 \cos \pi + \cos 2\pi]}{-\pi}$$

$$= \frac{[2 \cdot 1 + 1] - [2 \cdot (-1) + 1]}{-\pi}$$

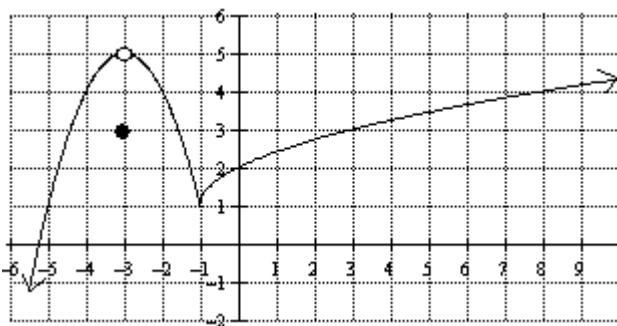
$$= \frac{3 - [-1]}{-\pi}$$

$$-2 \sin c - 2 \sin(2c) = \frac{4}{-\pi}$$

$$c = 0.517$$

$$c = 1.748$$

Using the graph of the function, $f(x)$, pictured below, and given the intervals in the table below, determine if Rolle's or Mean Value Theorem, whichever is indicated, can be applied or not. Give SPECIFIC reasons for your answers.



10. $[-5, -1]$ Rolle's Theorem	<p>a) $f(x)$ is not continuous on $[-5, -1]$ due to point discontinuity at $x = -3$ \therefore Rolle's Theorem does not apply.</p>
11. $[-2, 8]$ Rolle's Theorem	<p>b) $f(x)$ is not differentiable on $(-2, 8)$ due to a cusp at $x = -1$. \therefore Rolle's Theorem does not apply.</p>
12. $[-1, 8]$ Mean Value Theorem	<p>a) $f(x)$ is continuous on $[-1, 8]$ b) $f(x)$ is differentiable on $(-1, 8)$ \therefore The Mean Value Theorem applies.</p>

