NO CALCULATOR PERMITTED

A particle moves along the x – axis so that its position at any time $t \geq 0$ is given by the function $p(t) = t^3 - 4t^2 - 3t + 1$, where p is measured in feet and t is measured in seconds.

a. Find the average velocity on the interval $t = 1$ and $t = 2$ seconds. Give your answer using correct units.

b. On what interval(s) of time is the particle moving to the left? Justify your answer.

c. Using appropriate units, find the value of $p'(3)$ and $p''(3)$. Based on these values, describe the motion of the particle at $t = 3$ seconds. Give a reason for your answer.

d. What is the maximum velocity on the interval from $t = 1$ to $t = 3$ seconds. Show the analysis that leads to your conclusion.

e. Find the total distance that the particle moves on the interval [1, 5]. Show and explain your analysis.

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a) Average velocity $= \dfrac{p(1)-p(2)}{1-2} = \dfrac{\left[(1)^3-4(1)^2-3(1)+1\right]-\left[(2)^3+4(2)^2-3(2)+1\right]}{-1}$

$= \dfrac{[1-4-3+1]-[8+16-6+1]}{-1} = \dfrac{[-5]-[19]}{-1} = \dfrac{-24}{-1}$

$= 24$

– 8 ft/sec

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b) $v = 3t^2 - 8t - 3$

$0 = \dfrac{(3t-9)(3t+1)}{3}$

$0 = (t-3)(3t+1)$

$0 = t-3$, $3 = t$; $3t+1=0$, $3t=-1$, $t = \cancel{-1/3}$

NEG POS

v: 0 —— ($t=1$) —— 3 —— ($t=4$) →

The particle is moving left on $(0,3)$ b/c $v(t)<0$ on $(0,3)$

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c) $p''(t) = a(t) = 6t-8$

$p'(3) = v(3) = [(3)-3][3(3)+1] = 0$ feet per second

$p''(3) = a(3) = 6(3)-8 = 10$ feet per second^2

The particle is changing direction because the $v(t)=0$ at $t=3$ and $a(t)>0$ at $t=3$

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d) $v(1) = [(1)-3][3(1)+1] = [-2][4] = -8$ ft/sec

$v(3) = [(3)-3][3+1] = [0][4] = 0$ ft/sec

Since $p(t)$ is differentiable on $(1,3)$ thus continuous on $[1,3]$, The EVT applies.

The maximum velocity is 8 feet/sec moving left.

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e) Total Distance $= |p(1)-p(3)| + |p(3)-p(5)|$

$= |(-5)-(-17)| + |(-17)-(11)|$

$= |12| + |-28|$

$= 12+28$

$= 40$ feet

$p(3) = 3^3 - 4(3)^2 - 3(3) + 1$
$= 27 - 36 - 9 + 1$
$= 28 - 45$
$p(3) = -17$

$p(5) = 5^3 - 4(5)^2 - 3(5) + 1$
$= 125 - 100 - 15 + 1$
$= 126 - 115$
$p(5) = 11$

$p(1) = 1^3 - 4(1)^2 - 3(1) + 1$
$= 1 - 4 - 3 + 1$
$p(1) = -5$

A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes, where v is a differentiable function of t. Selected values of $v(t)$ for $0 \le t \le 40$ are shown in the table below

t (min)	0	5	10	15	20	25	30	35	40
$v(t)$ (miles per min)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

a. Find the average acceleration on the interval $5 \le t \le 20$. Express your answer using correct units of measure.

b. Based on the values in the table, on what interval(s) is the acceleration of the plane guaranteed to equal zero on the open interval $0 < t < 40$? Justify your answer.

c. Does the data represent velocity values of the plane moving away from its point of origin or returning to its point of origin? Give a reason for your answer.

d. The function f, defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \le t \le 40$. According to this model, what is the acceleration of the plane at $t = 23$? What does this value indicate about the velocity at $t = 23$? Justify your answer, indicating units of measure.

a) Average acceleration $= \dfrac{v(5)-v(20)}{5-20} = \dfrac{9.2-4.5}{-15} = \dfrac{4.7}{-15} \approx -0.313$ miles/minute^2

b) $a(t) = v'(t) = 0$ ...

Rolles Theorem guarantees a value of c on $(0,15)$ and $(25,30)$ such that $v'(t)=0$

b/c (a) $v(t)$ is continuous on $[0,15]$ and $[25,30]$

(b) $v(t)$ is differentiable on $(0,15)$ and $(25,30)$

(c) $f(0) = f(15)$ and $f(25) = f(30)$

c) The data represents velocity values of the plane moving away from its point of origin b/c $v(t) > 0$ for each t in the table.

d) $f(t) = 6 + \cos\left(\frac{1}{10}t\right) + 3\sin\left(\frac{7}{40}t\right)$

$a(t) = f'(t) = -\frac{1}{10}\sin\left(\frac{1}{10}t\right) + \frac{21}{40}\cos\left(\frac{7}{40}t\right)$

$f'(23) = -\frac{1}{10}\sin\left(\frac{23}{10}\right) + \frac{21}{40}\cos\left(\frac{161}{40}\right)$

$f'(23) \approx -0.408$ miles/min^2

At 23 minutes, the plane's velocity is decreasing by 0.408 miles per minute^2. The velocity is decreasing because $a(t) < 0$ at $t = 23$.