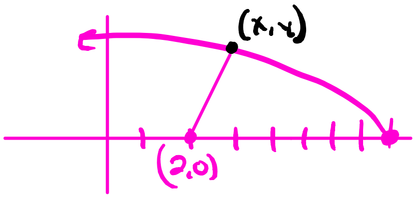
Name____Date____________________Class__________

$-x+8>0$
$8>x$

Homework 5.6

1. Find the point on the graph of $f(x)=\sqrt{-x+8}$ so that the point (2, 0) is closest to the graph.



① $d=\sqrt{[\Delta x]^2+[\Delta y]^2}$

$d=\sqrt{[x-2]^2+[y-0]^2}$

② $\boxed{y=\sqrt{-x+8}}$

③ $d=\sqrt{[x-2]^2+[\sqrt{-x+8}]^2}$

$d=\sqrt{x^2-4x+4+-x+8}$

$d=(x^2-5x+12)^{1/2}$

④ $d'=\frac{1}{2}(x^2-5x+12)^{-1/2}(2x-5)$

$d'=\dfrac{2x-5}{2(x^2-5x+12)^{1/2}}$

$d'=0$	$d'=\text{und}$
$2x-5=0$	$2(x^2-5x+12)^{1/2}=0$
$2x=5$	$x^2-5x+12=0$
$x=5/2$	Disc $=b^2-4ac$
	$=(-5)^2-4(1)(12)$
	$=25-48$
	Disc $=-23$
	No Solution

d': NEG ($x=0$) | $5/2$ | POS ($x=5$) | 8

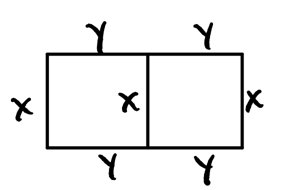
Since d' changes from + to − at $x=5/2$, the distance is a minimum.

⑤ $f(5/2)=\sqrt{-5/2+\frac{16}{2}}$

$f(5/2)=\sqrt{\frac{11}{2}}$

Point: $\left(\frac{5}{2},\sqrt{\frac{11}{2}}\right)$ is the closest to (2,0)

2. A rancher has 200 total feet of fencing with which to enclose two adjacent rectangular corrals. What dimensions should each corral be so that the enclosed area will be a maximum?



x = length (ft)
y = width (ft)
A = Area (ft²)
F = Fence perimeter

① $A=x\cdot 2y$

③ $A=\cancel{2}x\cdot\left(\dfrac{200-3x}{\cancel{4}\,2}\right)$

$A=\dfrac{200x-3x^2}{2}$

$A=100x-\frac{3}{2}x^2$

② $F=3x+4y$

$200=3x+4y$

$200-3x=4y$

$\dfrac{200-3x}{4}=y$

$\dfrac{200-3\left(\frac{100}{3}\right)}{4}=y$

$\dfrac{200-100}{4}=y$

$\dfrac{100}{4}=y$

$25=y$

④ $A'=100-3x$

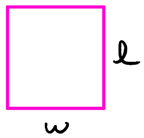
$A'=0$	$A'=\text{und}$
$100-3x=0$	None
$100=3x$	
$\frac{100}{3}=x$	

CANDIDATE'S TEST

x	A(x)
0	0
100/3	1666.667
200/3	0

⑤ The dimension of each corrals should be 25 feet by $\frac{100}{3}$ feet where the shared side is $\frac{100}{3}$ feet.

3. The area of a rectangle is 64 square feet. What dimensions of the rectangle would give the smallest perimeter?



(1) $P = 2w + 2y$

(2) $A = wl$

$64 = wl$

$\frac{64}{w} = l$

$\frac{64}{8} = l$

$8 = l$

(3) $P = 2w + 2\left(\frac{64}{w}\right)$

$P = 2w + 128w^{-1}$

(4) $P' = 2 - 128w^{-2}$

$P' = \frac{2w^2 - 128}{w^2}$

$P' = \frac{2(w-8)(w+8)}{w^2}$

$P' = 0$	$P' = \text{und}$
$w - 8 = 0$ | $w + 8 = 0$	$w^2 = 0$
$w = \cancel{-8}, 8$	$w = 0$

NEG POS

P' : 0 —— ($x=1$) —— 8 —— ($x=100$) ——→

P' changes from $-$ to $+$ at $x = 8$, so perimeter is MIN.

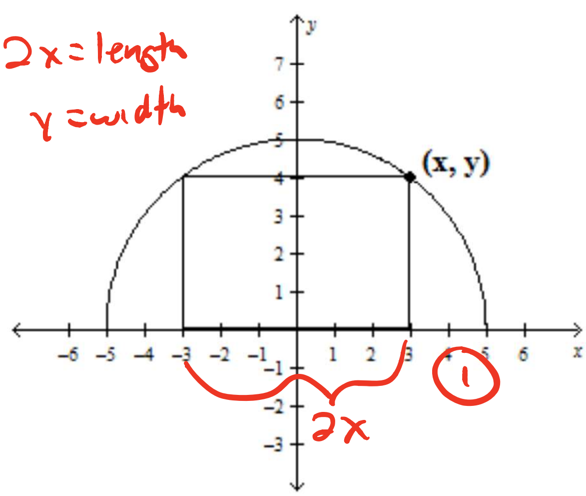
(5) The rectangle should have dimensions 8 feet by 8 feet.

4. A rectangle is bound by the x – axis and the graph of a semicircle defined by $y = \sqrt{25 - x^2}$. What length and width should the rectangle have so that its area is a maximum?

(2)

$2x = \text{length}$

$y = \text{width}$



(1) $A = 2xy$

(3) $A = 2x(25 - x^2)^{1/2}$

(4) $A' = 2 \cdot (25 - x^2)^{\frac{1}{2}} + 2x \cdot \frac{1}{2}(25 - x^2)^{-\frac{1}{2}}(-2x)$

$A' = 2(25 - x^2)^{\frac{1}{2}} - 2x^2(25 - x^2)^{-1/2}$

$A' = 2(25 - x^2)^{-1/2}\left[(25 - x^2) - x^2\right]$

$A' = \frac{2}{\sqrt{25 - x^2}}(25 - 2x^2)$

$A' = \frac{2(25 - 2x^2)}{\sqrt{25 - x^2}}$

$A' = 0$	$A' = \text{und}$
$0 = 2(25 - 2x^2)$	$\sqrt{25 - x^2} = 0$
$0 = 25 - 2x^2$	$25 - x^2 = 0$
$2x^2 = 25$	$(5 - x)(5 + x) = 0$
$x^2 = \frac{25}{2}$	$x = 5, x = -5$
$x = \cancel{-\sqrt{\frac{25}{2}}}, \sqrt{\frac{25}{2}}$	

$\text{length} = 2\left(\sqrt{\frac{25}{2}}\right)$

$\text{width} = \sqrt{25 - \left(\sqrt{\frac{25}{2}}\right)^2}$

$\text{width} = \sqrt{25 - \frac{25}{2}}$

POS NEG

A' : -5 —— ($x=0$) —— $\sqrt{\frac{25}{2}}$ —— ($x=4$) —— 5

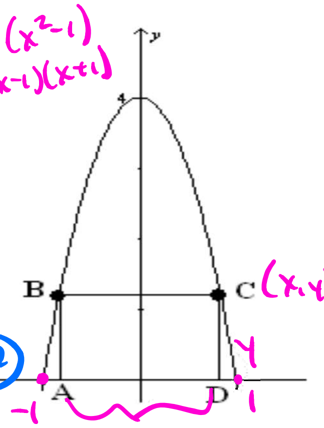
A' changes from $+$ to $-$ at $x = \sqrt{\frac{25}{2}}$, so Area is a MAX

(5) The length should be $2\sqrt{\frac{25}{2}}$ and the width is $\sqrt{25 - \frac{25}{2}}$ to maximize Area.

5. A rectangle ABCD with sides parallel to the coordinate axes is inscribed in the region enclosed by the graph of $y = -4x^2 + 4$ as shown in the figure below. Find the x and y coordinates of the point C so that the area of the rectangle is a maximum.

① $0 = -4(x^2-1)$

$0 = -4(x-1)(x+1)$



②

③ $A = 2xy$

$A = 2x(-4x^2+4)$

$A = -8x^3 + 8x$

$A' = -24x^2 + 8$

$A' = -24(x^2 - \frac{1}{3})$

$A' = -24(x - \sqrt{\frac{1}{3}})(x + \sqrt{\frac{1}{3}})$

$A' = 0$	$A' =$ und
$0 = -24(x-\sqrt{\frac{1}{3}})(x+\sqrt{\frac{1}{3}})$	never
$x = \cancel{-\sqrt{\frac{1}{3}}}, \sqrt{\frac{1}{3}}$	

④ A': POS ($x=0$) on $(-1, \sqrt{\frac{1}{3}})$, NEG ($x = 3/4$) on $(\sqrt{\frac{1}{3}}, 1)$

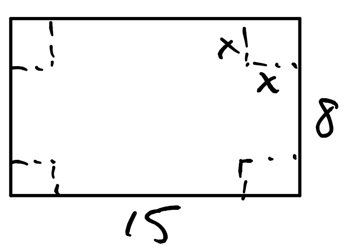
⑤ [A' changes from + to − at $x = \sqrt{\frac{1}{3}}$ so Area is maximized.]

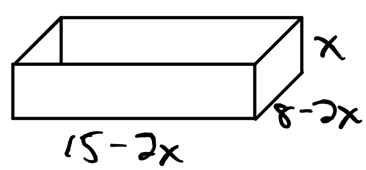
⑥ $y = -4x^2 + 4$

$y = -4(\sqrt{\frac{1}{3}})^2 + 4$

⑦ $\boxed{\text{The point } \left(\sqrt{\tfrac{1}{3}}, -4\left(\sqrt{\tfrac{1}{3}}\right)^2 + 4\right) \text{ maximizes the area}}$

6. Find the maximum volume of a box that can be made by cutting squares from the corners of an 8 inch by 15 inch rectangular sheet of cardboard and folding up the sides. $0 < x < 4$





$x =$ length of cut (in)

$V =$ volume

① $V = (15-2x)(8-2x)x$

$V = (120 - 16x - 30x + 4x^2)x$

$V = 120x - 46x^2 + 4x^3$

$V = 4x^3 - 46x^2 + 120x$

④ $V' = 12x^2 - 92x + 120$

$V' = 4(3x^2 - 23x + 30)$

$V' = 4(3x-5)(x-6)$

$V' = 0$	$V' =$ und
$x = \cancel{6}, 5/3$	never

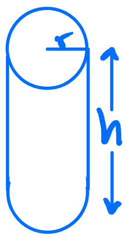
m $= 90x^2$
A $= -23x$
N $= -5x, -18x$
$3x^2 - 23x + 30$
$3x^2 - 5x - 18x + 30$
$x(3x-5) - 6(3x-5)$
$(3x-5)(x-6)$

V': POS ($x=1$) on $(0, 5/3)$, NEG ($x=5$) on $(5/3, 6)$

[V' changes from + to − at $x = 5/3$ so Volume is maximized.]

⑤ $\boxed{V = [15 - 2(5/3)][8 - 2(5/3)](5/3) \text{ in}^3 \text{ is the maximum volume.}}$

7. The volume of a cylindrical tin can with a top and bottom is to be 16π cubic inches. If a minimum amount of tin is to be used to construct the can, what must the height, in inches, of the can be?



① r = radius (in)
h = height (in)
V = volume (in^3)
SA = Surface Area (in^2)

② $V = A_{base} \cdot h$

$16\pi = \pi r^2 \cdot h$

$\frac{16}{r^2} = h$

③ $SA = 2A_{base} + A_{lateral}$

$SA = 2\pi r^2 + 2\pi r \cdot h$

④ $SA = 2\pi r^2 + 2\pi r\left(\frac{16}{r^2}\right)$

$SA = 2\pi r^2 + 32\pi r^{-1}$

$SA' = 4\pi r - 32\pi r^{-2}$

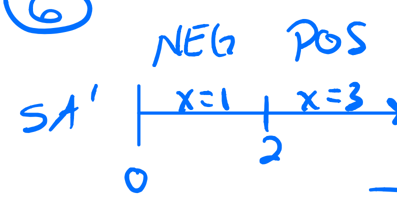
$SA' = \frac{4\pi r^3 - 32\pi}{r^2}$

$SA' = \frac{4\pi(r^3 - 8)}{r^2}$

⑤

$SA' = 0$	$SA' = \text{und}$
$4\pi(r^3 - 8) = 0$	$r^2 = 0$
$r^3 - 8 = 0$	$r = 0$
$r^3 = 8$	
$r = 2$	

⑥



SA' changes from − to + at $x = 2$ so Surface Area is minimized.

⑦ $\frac{16}{2^2} = h$

$\frac{16}{4} = h$

$4 = h$

The height must be 4 inches to minimize amount of tin.