Homework 5.7

AP-Style questions….

1. $\lim_{x \to \frac{\pi}{2}} \frac{3\cos x}{2x - \pi}$ is

   a. $-\frac{3}{2}$
   b. 0
   c. $\frac{3}{2}$
   d. nonexistent

$\lim_{x \to \frac{\pi}{2}} (3\cos x) = 3 \cdot \cos(\frac{\pi}{2}) = 3 \cdot 0 = 0$

$\lim_{x \to \pi/2} (2x - \pi) = 2(\frac{\pi}{2}) - \pi = \pi - \pi = 0$

This limit produces the indeterminate form $\frac{0}{0}$

L'HOSPITAL'S

$\lim_{x \to \frac{\pi}{2}} \frac{3\cos x}{2x - \pi} = \lim_{x \to \pi/2} \frac{-3\sin x}{2} = \frac{-3\sin(\pi/2)}{2} = \frac{-3(1)}{2} = -3/2$

2. $\lim_{x \to 0} \frac{6e^{4x} - 2e^{3x} - 4}{\sin(2x)} =$

   a. 2
   b. 4
   c. 9
   d. 18

$\lim_{x \to 0} (6e^{4x} - 2e^{3x} - 4) = 6e^{4 \cdot 0} - 2e^{3 \cdot 0} - 4 = 6 \cdot 1 - 2 \cdot 1 - 4 = 0$

$\lim_{x \to 0} (\sin(2x)) = \sin(2 \cdot 0) = \sin 0 = 0$

This limit produces the indeterminate form $\frac{0}{0}$

L'HOSPITAL'S

$\lim_{x \to 0} \frac{6e^{4x} - 2e^{3x} - 4}{\sin(2x)} = \lim_{x \to 0} \frac{24e^{4x} - 6e^{3x}}{\underbrace{2 \cdot \cos(2x)}_{\text{chain}}} = \frac{24e^{4 \cdot 0} - 6e^{3 \cdot 0}}{2\cos(2 \cdot 0)} = \frac{24 \cdot 1 - 6 \cdot 1}{2\cos(0)} = \frac{18}{2 \cdot 1} = 9$

3. Let f be the function defined by $f(x) = 2x + 3e^{-5x}$, and let g be a differentiable function with derivative given by $g'(x) = \frac{1}{x} + 4\cos\left(\frac{5}{x}\right)$. It is known that $\lim_{x \to \infty} g(x) = \infty$. The value of $\lim_{x \to \infty} \frac{f(x)}{g(x)}$ is

   a. 0
   b. ½
   c. 1
   d. nonexistent

$\lim_{x \to \infty} \frac{f(x)}{g(x)}$

$\lim_{x \to \infty} (2x + 3e^{-5x}) = \lim_{x \to \infty} (2x + \frac{3}{e^{5x}}) = \infty + 0 = \infty$

$\lim_{x \to \infty} g(x) = \infty$

This limit produces the indeterminate form $\frac{\infty}{\infty}$

L'HOSPITAL'S

$\lim_{x \to \infty} \frac{f(x)}{g(x)} = \lim_{x \to \infty} \frac{2 - 15e^{-5x}}{\frac{1}{x} + 4\cos(\frac{5}{x})} = \lim_{x \to \infty} \frac{2 - \frac{15}{e^{5x}}}{\frac{1}{x} + 4\cos(\frac{5}{x})} = \frac{2 - 0}{0 + 4\cos(0)} = \frac{2}{4 \cdot 1}$

4. $\lim_{x\to\pi} \frac{x+\pi\sec x}{x^2-\pi^2}$ is

$$\begin{cases} \lim_{x\to\pi} (x+\pi\sec x) = \pi + \pi\sec(\pi) = \pi + \pi(-1) = 0 \\ \lim_{x\to\pi} (x^2-\pi^2) = \pi^2 - \pi = 0 \end{cases}$$

a. $-\frac{\pi}{2}$

b. 0

c. $\frac{1}{2\pi}$

d. nonexistent

This limit produces the indeterminate form $\frac{0}{0}$

L'HOSPITALS

$$\lim_{x\to\pi} \frac{x+\pi\sec x}{x^2-\pi^2} = \lim_{x\to\pi} \frac{1+\overbrace{\pi\sec x\tan x}^{(\pi\sec x)'}}{2x} = \frac{1+\pi\sec\pi\cdot\tan\pi}{2\pi} = \frac{1+\pi(-1)\cdot 0}{2\pi} = \frac{1}{2\pi}$$

5. $\lim_{t\to 0} \frac{\sin t}{\ln(2e^t-1)} =$

$$\begin{cases} \lim_{t\to 0} (\sin t) = \sin 0 = 0 \\ \lim_{t\to 0} (\ln(2e^t-1)) = \ln(2e^0-1) = \ln(2\cdot 1) = \overbrace{\ln 1}^{\log_e 1 \Rightarrow e^? = 1} = 0 \end{cases}$$

a. -1

b. 0

c. ½

d. 1

This limit produces the indeterminate form $\frac{0}{0}$

L'HOSPITALS

$$\lim_{t\to 0} \frac{\sin t}{\ln(2e^t-1)} = \lim_{t\to 0} \frac{\cos t}{\underbrace{\frac{2e^t}{2e^t-1}}_{[\ln f(x)]' = \frac{f'(x)}{f(x)}}} = \frac{\cos(0)}{\frac{2e^0}{2e^0-1}} = \frac{1}{\frac{2\cdot 1}{2\cdot 1-1}} = \frac{1}{\frac{2}{1}} = \frac{1}{2}$$

6. Let f be the function defined by $f(x) = 3x + 2e^{-3x}$, and let g be a differentiable function with derivative given by $g'(x) = 4 + \frac{1}{x}$. It is known that $\lim_{x\to\infty} g(x) = \infty$. The value of $\lim_{x\to\infty} \frac{f(x)}{g(x)}$ is

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} \quad \begin{cases} \lim_{x\to\infty} f(x) = \lim_{x\to\infty} \left(3x + \frac{2}{e^{3x}}\right) = \infty + 0 = \infty \\ \lim_{x\to\infty} g(x) = \infty \end{cases}$$

a. 0

b. $\frac{3}{4}$

c. 1

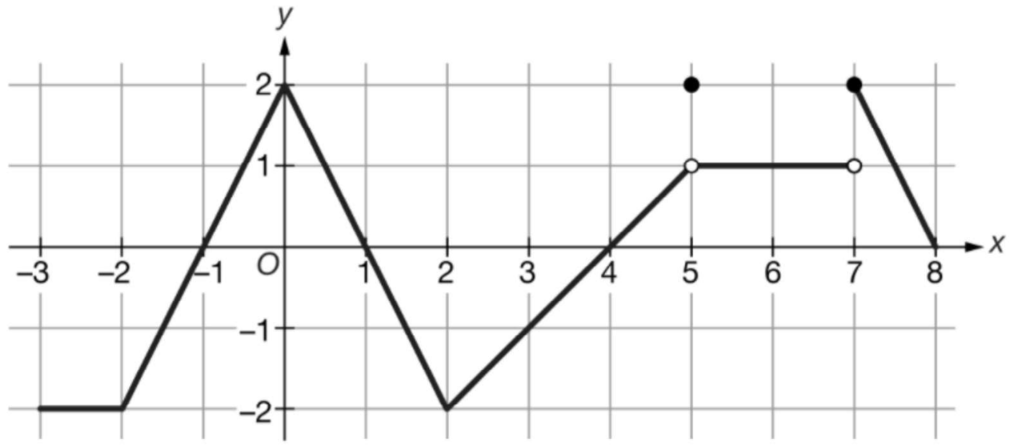
d. nonexistent

This limit produces the indeterminate form $\frac{\infty}{\infty}$

L'HOSPITALS

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{3-6e^{-3x}}{4-\frac{1}{x}} = \lim_{x\to\infty} \frac{3-\frac{6}{e^{3x}}}{4-\frac{1}{x}} = \frac{3-0}{4-0} = \frac{3}{4}$$

7. Unless otherwise specified, the domain of a function f is assumed to be set of all real numbers x for which $f(x)$ is a real number.



Graph of f

The graph of the function f on the closed interval $-3 \le x \le 8$ consists of six line segments and the point $(5, 2)$ as shown in the figure. The function g is given by $g(x) = \frac{1}{10}(4x^3 + 3x^2 - 10x - 17)$.

Find $\lim\limits_{x\to 1} \frac{f(x)}{g(x)+2}$. Show the work that leads to your answer.

See Rubric

8. A particle moves along the x-axis so that its position at time t is given by $x(t) = \frac{\sin(\pi t)}{2-t}$ for all times $t \neq 2$. As time *t* approaches *2*, what is the limit of the position of the particle? Show the work that leads to your answer.

See Rubric

Key 5.7

1. A
2. C
3. B
4. C
5. C
6. B

7.

The student response accurately includes all three of the criteria below.

☐ L'Hospital's Rule

☐ $g'(x)$

☐ answer

Solution:

$\lim_{x\to 1} f(x) = 0$ and $\lim_{x\to 1}(g(x)+2) = -2+2 = 0.$

Using L'Hospital's Rule,

$$\lim_{x\to 1}\frac{f(x)}{g(x)+2} = \lim_{x\to 1}\frac{f'(x)}{g'(x)} = \lim_{x\to 1}\frac{f'(x)}{\frac{1}{10}(12x^2+6x-10)}$$

$$= \frac{-2}{\frac{1}{10}(12+6-10)} = \frac{-20}{8} = -\frac{5}{2}.$$

8.

The student response accurately includes both of the criteria below.

☐ application of L'Hospital's Rule

☐ answer

Solution:

$\lim_{t\to 2}\sin(\pi t) = 0$

$\lim_{t\to 2}(2-t) = 0$

By L'Hospital's Rule,

$$\lim_{t\to 2}\frac{\sin(\pi t)}{2-t} = \lim_{t\to 2}\frac{\pi\cos(\pi t)}{-1} = -\pi$$