

Homework 6.1

For problems 1 – 12, find the indefinite integrals below.

$$1. \int (\sqrt[3]{x} + 3) dx = \int (x^{1/3} + 3) dx \\ = \frac{3}{4}x^{4/3} + 3x + C$$

$$2. \int x^2(2x^2 + 3x) dx = \int (2x^4 + 3x^3) dx \\ = \frac{2}{5}x^5 + \frac{3}{4}x^4 + C$$

$$3. \int \left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right) dx = \int (x^{1/2} + \frac{1}{2}x^{-1/2}) dx \\ = \frac{2}{3}x^{3/2} + x^{1/2} + C$$

$$4. \int y^3 \sqrt{y} dy = \int y^{7/2} dy \\ = \int y^{7/2} dy \\ = \frac{2}{9}y^{9/2} + C$$

$$5. \int (2x - 3x^2) dx = x^2 - x^3 + C$$

$$6. \int (x^{3/2} + 2x + 1) dx = \frac{2}{5}x^{5/2} + x^2 + x + C$$

$$7. \int \frac{3x^2 - 2x + 3}{x^3} dx = \int (3x^{-1} - 2x^{-2} + 3x^{-3}) dx \\ = 3\ln|x| + 2x^{-1} - \frac{3}{2}x^{-2} + C$$

$$8. \int \frac{1}{w\sqrt{w}} dw = \int w^{-1} w^{-1/2} dw \\ = \int w^{-3/2} dw \\ = -2w^{-1/2} + C$$

$$9. \int \frac{x^3+3}{\sqrt{x}} dx = \int (x^{4/2} + 3)x^{-1/2} dx$$

$$= \int [x^{4/2} + 3x^{-1/2}] dx$$

$$= \frac{2}{7}x^{7/2} + 6x^{1/2} + C$$

$$10. \int (\theta^2 + \cos \theta) d\theta = \frac{1}{3}\theta^3 + \sin \theta + C$$

$$11. \int (x+3)(x-3)^2 dx = \int (x+3)(x^2 - 6x + 9) dx$$

$$= \int (x^3 - 6x^2 + 9x + 3x^2 - 18x + 27) dx$$

$$= \int (x^3 - 3x^2 - 9x + 27) dx$$

$$= \frac{1}{4}x^4 - x^3 - \frac{9}{2}x^2 + 27x + C$$

$$12. \int (\sqrt{x} - \sin x + 2) dx$$

$$= \frac{2}{3}x^{3/2} + \cos x + 2x + C$$

For problems 13 and 14, find the indicated function based on the given information.

13. If $f'(x) = 2x - \sin x$ and $f(0) = 4$, find $f(x)$.

$$f(x) = \int (2x - \sin x) dx = x^2 + \cos x + C$$

$$f(0) = 4$$

$$4 = (0)^2 + \cos(0) + C$$

$$4 = 0 + 1 + C$$

$$3 = C$$

$$\therefore f(x) = x^2 + \cos x + 3$$

14. If $f''(x) = x^2$, $f'(0) = 6$, and $f(0) = 3$, find $f(x)$.

$$f'(x) = \int x^2 dx = \frac{1}{3}x^3 + C$$

$$f'(0) = 6$$

$$6 = \frac{1}{3}(0)^3 + C$$

$$6 = C$$

$$f'(x) = \frac{1}{3}x^3 + 6$$

$$f(x) = \int (\frac{1}{3}x^3 + 6) dx = \frac{1}{12}x^4 + 6x + C$$

$$f(0) = 3$$

$$3 = \frac{1}{12}(0)^4 + 6(0) + C$$

$$3 = C$$

$$\therefore f(x) = \frac{1}{12}x^4 + 6x + 3$$