

**Homework 6.4**

Given  $\int_2^6 f(x)dx = 10$  and  $\int_2^6 g(x)dx = -2$ , find the values of each of the following definite integrals, if possible, by rewriting the given integral using the properties of integrals.

<p>1. <math>\int_2^6 [f(x) + g(x)]dx</math></p> $= \int_2^6 f(x)dx + \int_2^6 g(x)dx$ $= 10 + (-2)$ $= 8$	<p>2. <math>\int_2^6 [2f(x) - 3g(x)]dx</math></p> $= 2 \int_2^6 f(x)dx - 3 \int_2^6 g(x)dx$ $= 2(10) - 3(-2)$ $= 20 + 6$ $= 26$	<p>3. <math>\int_2^6 [2x + 2g(x)]dx = \int_2^6 2x dx + 2 \int_2^6 g(x)dx</math></p> $= x^2 \Big _2^6 + 2 \cdot (-2)$ $= [6^2 - 2^2] - 4$ $= [36 - 4] - 4$ $= 28$
---	---	--

Given  $\int_{-2}^4 f(x)dx = -6$  and  $\int_{-2}^4 g(x)dx = 4$ , find the values of each of the following definite integrals.

Rewrite the given integral using the properties of integrals. Then, find the value.

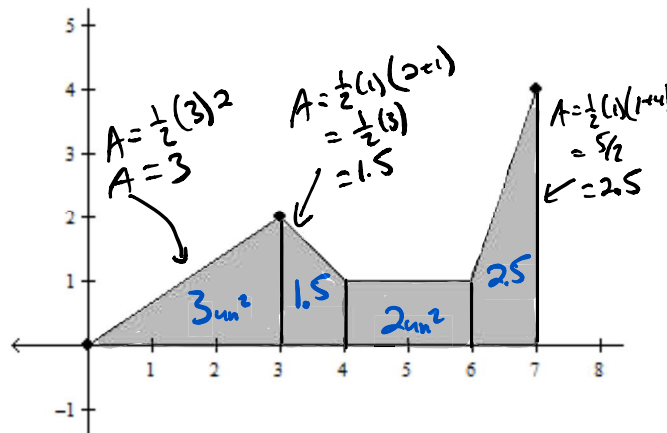
<p>4. <math>\int_{-2}^4 [f(x) + 4]dx = \int_{-2}^4 f(x)dx + \int_{-2}^4 4dx</math></p> $= -6 + 4x \Big _{-2}^4$ $= -6 + [4(4) - 4(-2)]$ $= -6 + 16 + 8$ $= 18$	<p>5. <math>\int_{-2}^4 [3g(x) + x]dx = 3 \int_{-2}^4 g(x)dx + \int_{-2}^4 x dx</math></p> $= 3 \cdot 4 + \frac{1}{2} x^2 \Big _{-2}^4$ $= 12 + \left[ \frac{1}{2} (4)^2 - \frac{1}{2} (-2)^2 \right]$ $= 12 + \left[ \frac{1}{2} (16) - \frac{1}{2} (4) \right]$ $= 12 + [8 - 2]$ $= 18$
<p>6. <math>\int_{-2}^4 \left[ \frac{1}{2} f(x) + 3x^2 \right] dx = \frac{1}{2} \int_{-2}^4 f(x)dx + 3 \int_{-2}^4 x^2 dx</math></p> $= \frac{1}{2} (-6) + x^3 \Big _{-2}^4$ $= -3 + [4^3 - (-2)^3]$ $= -3 + [64 - -8]$ $= 69$	

Pictured below is the graph of  $f'(x)$ , the first derivative of a function  $f(x)$ . Use the graph to answer the following questions 8–10.

Graph of  $f'(x)$

7. What is the value of  $\int_0^7 f'(x) dx$

$$\begin{aligned}
 &= \int_0^3 f'(x) dx + \int_3^4 f'(x) dx + \int_4^6 f'(x) dx + \int_6^7 f'(x) dx \\
 &= 3 + 1.5 + 2 + 2.5 \\
 &= 9
 \end{aligned}$$



8. If  $f(0) = -3$ , what is the value of  $f(3)$ ?

$$\begin{aligned}
 \int_0^3 f'(x) dx &= f(x) \Big|_0^3 \\
 3 &= f(3) - f(0) \\
 3 &= f(3) - (-3) \\
 3 &= f(3) + 3 \\
 0 &= f(3)
 \end{aligned}$$

9. If  $f(3) = -1$ , what is the value of  $f(7)$ ?

$$\begin{aligned}
 \int_3^7 f'(x) dx &= f(x) \Big|_3^7 \\
 6 &= f(7) - f(3) \\
 6 &= f(7) - (-1) \\
 6 &= f(7) + 1 \\
 5 &= f(7)
 \end{aligned}$$

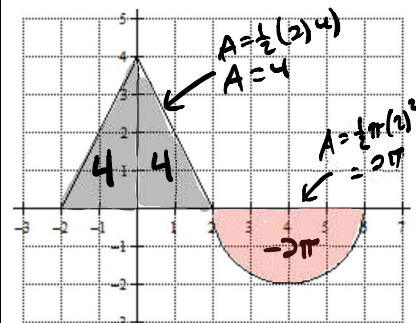
The graph of  $f'(x)$ , the derivative of a function,  $f(x)$ , is pictured below on the interval  $[-2, 6]$ . Write and find the value of a definite integral to find each of the indicated values of  $f(x)$  below. Also,  $f(-2) = 5$ .

10. Find the value of  $f(0)$ .

$$\begin{aligned}
 \int_{-2}^0 f'(x) dx &= f(x) \Big|_{-2}^0 \\
 4 &= f(0) - f(-2) \\
 4 &= f(0) - (5) \\
 9 &= f(0)
 \end{aligned}$$

11. Find the value of  $f(6)$ .

$$\begin{aligned}
 \int_{-2}^6 f'(x) dx &= f(x) \Big|_{-2}^6 \\
 8 - 2\pi &= f(6) - f(-2) \\
 8 - 2\pi &= f(6) - (5) \\
 13 - 2\pi &= f(6)
 \end{aligned}$$



Graph of  $f'$

$$\begin{aligned}
 A_0 &= \pi r^2 \\
 &= \pi(2)^2 \\
 &= 4\pi
 \end{aligned}$$